

Distributed Optimization and Learning for Aggregative Management of Energy Communities



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Distributed Algorithm for Coordination in Energy Communities

N interconnected units capable of

- Generating, consuming, and storing energy
- Exchanging energy and information

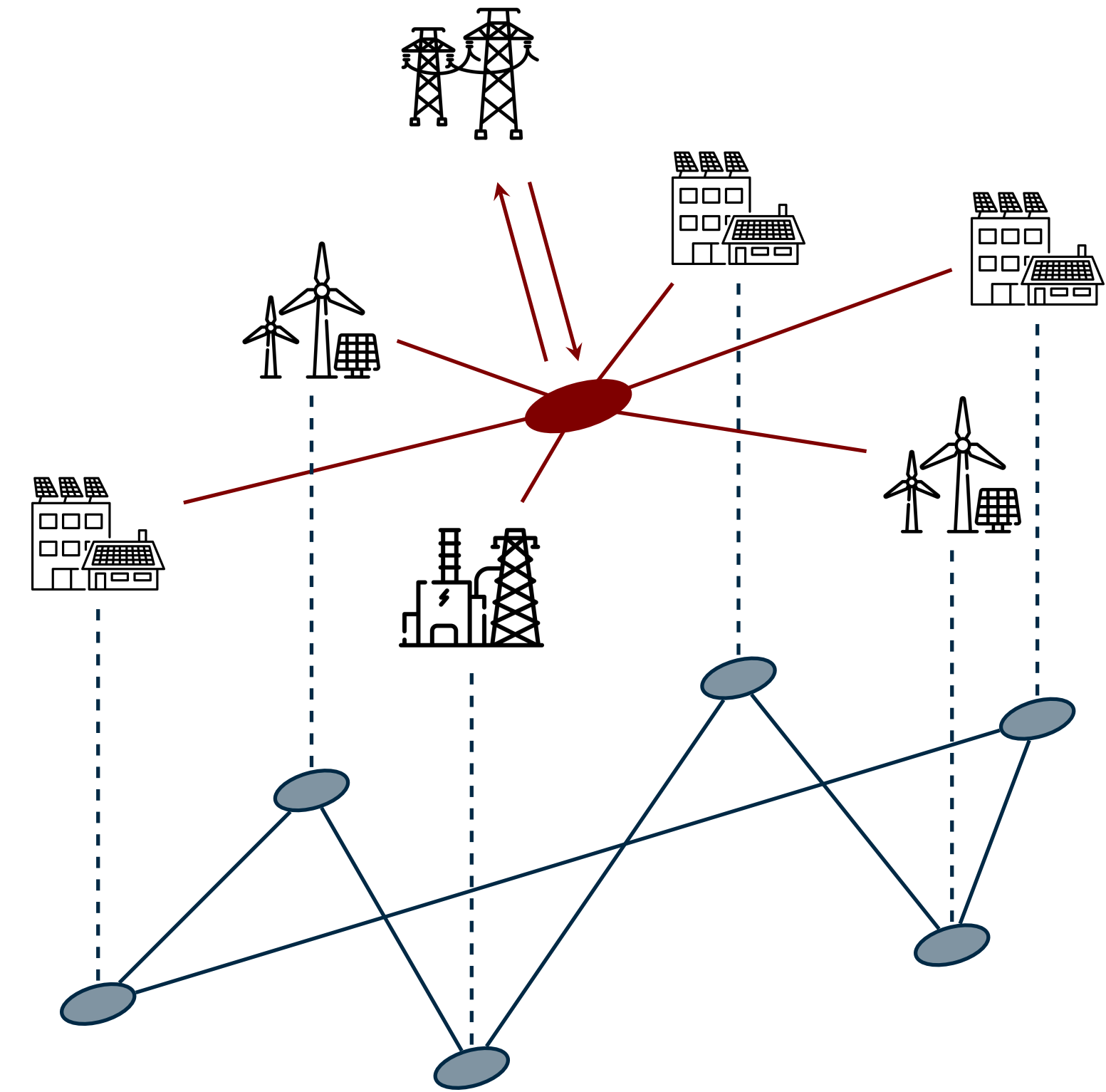
Energy exchange: (fully connected)

Units trade energy with everyone and the main grid

Information exchange: (graph-based)

Units communicate only with their neighbors

Goal: Leverage distributed optimization to enable cooperation among units, achieving a global objective



Energy Community Setup

Setup

N agents divided among *buyers* and *sellers*

Buyers: Can't satisfy their energy demand

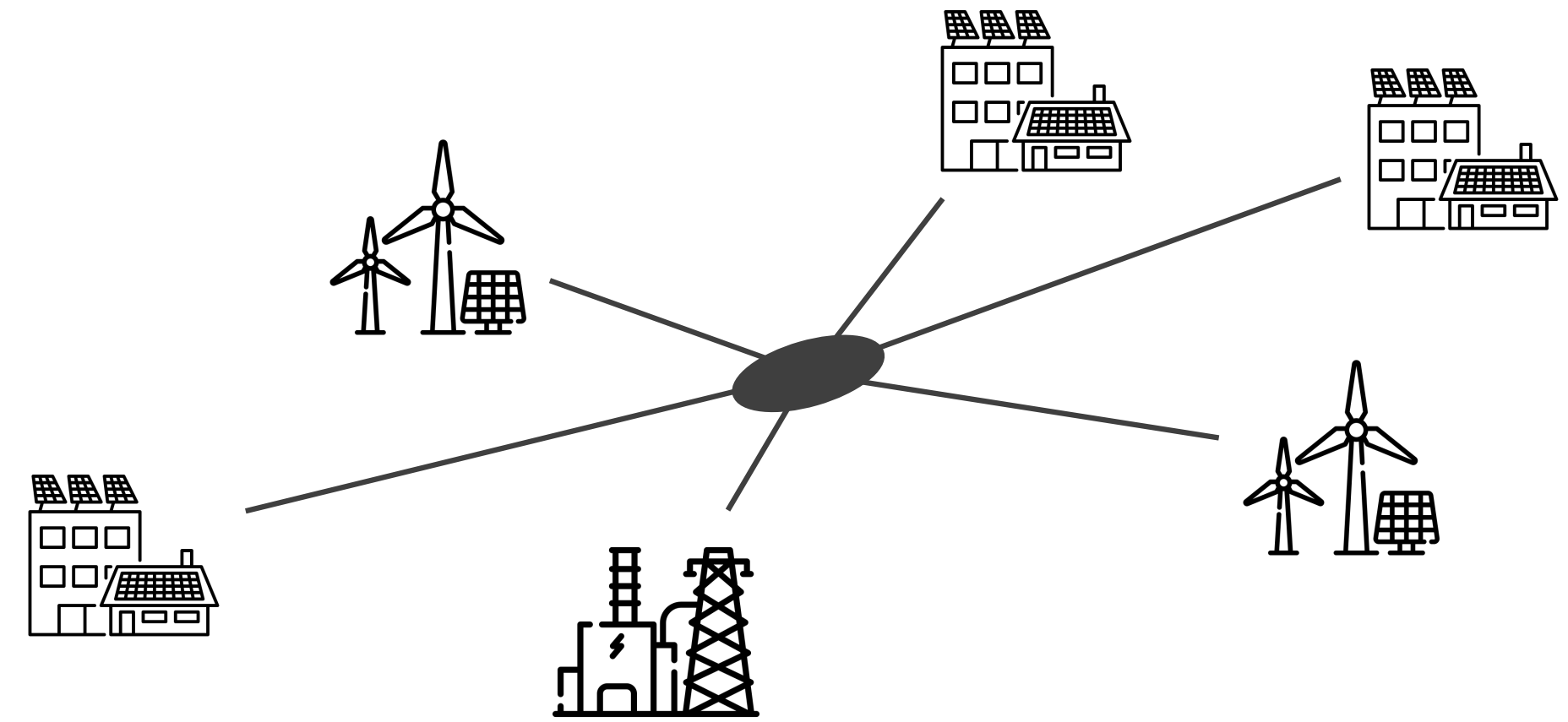
Goals:

- Demand satisfaction
- Buy energy at the lowest price

Sellers: Excess of energy that can be sold

Goals:

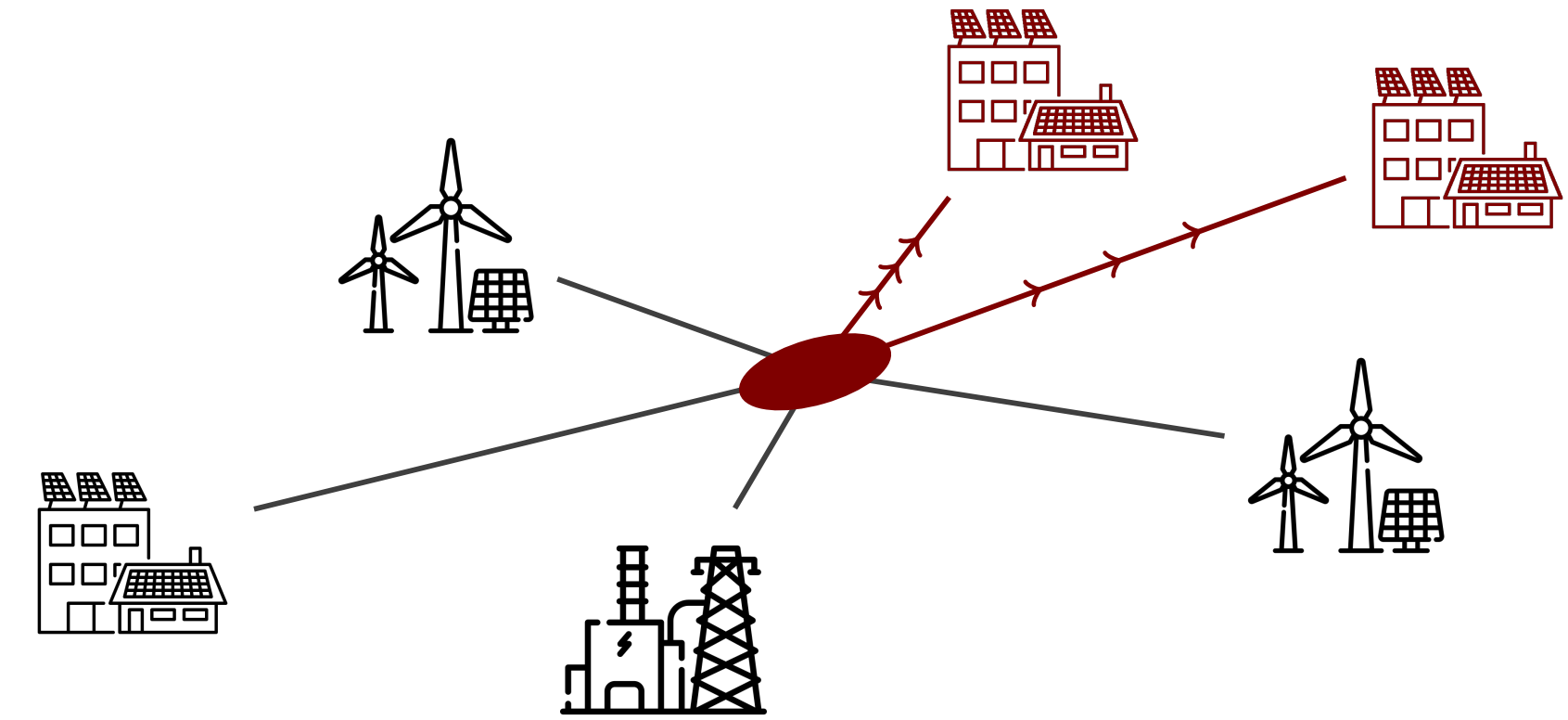
- Maximize revenues
- Store energy for future use



Buyer Detail

Decision Variable: $c_i \in [c_b, c_s]$

- Unit bid that buyer i is willing to pay
- Constrained by the main grid market prices



Energy received: follows a proportional sharing rule

$$E_i(c, w) = \hat{E}(w) \frac{c_i}{\sum_{i \in \mathcal{B}} c_i}$$

Note: $\hat{E}(w) := \sum_{j \in \mathcal{S}} \hat{E}_j w_j$ total energy sold, function of sellers decision variables and energy excesses \hat{E}_j

Cost function:

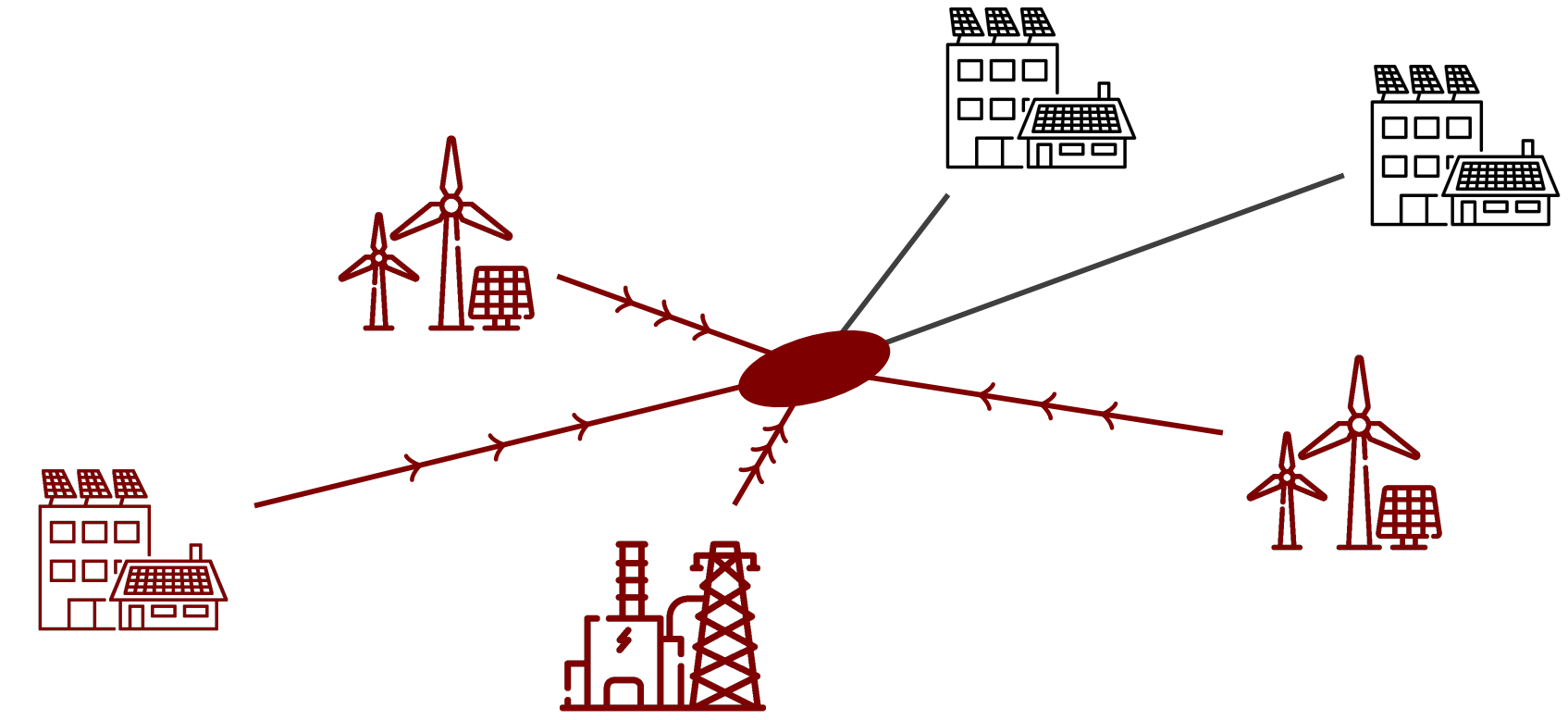
$$f_i^B := \underbrace{\epsilon_i^d \|E_i(c, w) - \bar{E}_i\|^2}_{\text{Demand satisfaction}} + \underbrace{\epsilon_i^b E_i(c, w)(c_i - c_s)}_{\text{Low energy cost}}$$

with \bar{E}_i the energy demand

Seller Detail

Decision Variable: $w_j \in [0,1]$

- Percentage of energy to sell
- Consistency constraints



Cumulative and Unit Revenue: follows a proportional sharing rule

$$R(c, w) := \sum_{i \in \mathcal{B}} c_i E_i(c, w) \quad \text{and} \quad R_j(c, w) := R(c, w) \frac{\hat{E}_j w_j}{\hat{E}(w)}$$

Cost function:

$$f_i^S(c, w) := \underbrace{\gamma_j^r R_j(c, w)}_{\text{Revenue maximisation}} - \underbrace{\gamma_j^s \ln \left(1 + \hat{E}_j (1 - w_j) \right)}_{\text{Satisfaction from storage}}$$

Distributed Aggregative Optimization Problem

Remark: f_i^B and f_j^S depends on an aggregation of all decision variables

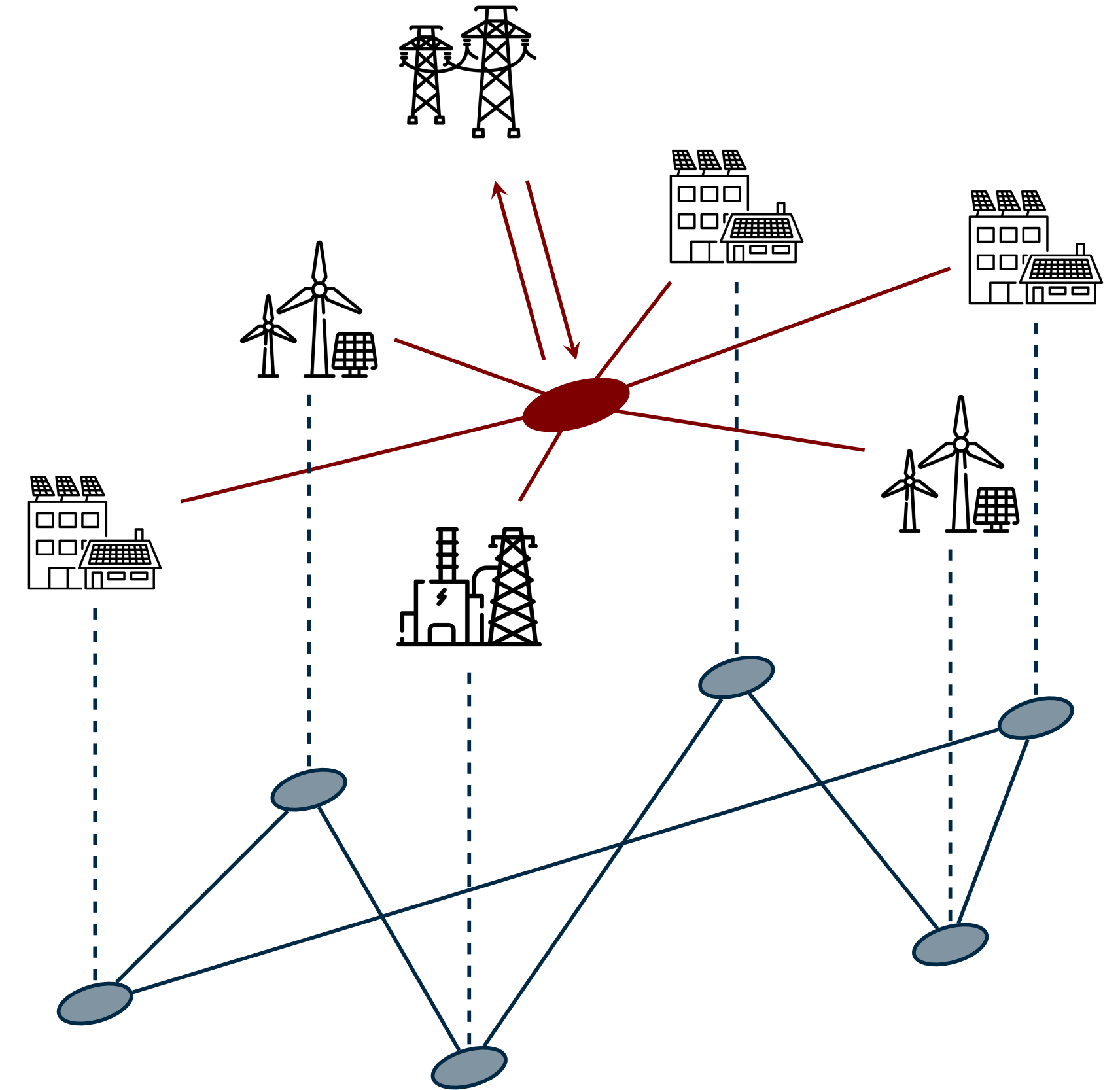
Optimization Problem:

$$\begin{aligned} \min_{\substack{c_1, \dots, c_I \\ w_{I+1}, \dots, w_J}} \quad & \sum_{i \in \mathcal{B}} f_i^B(c_i, \sigma(c, w)) + \sum_{j \in \mathcal{S}} f_j^S(w_j, \sigma(c, w)) \\ \text{subj. to} \quad & c_i \in [c_b, c_s], \quad \forall i \in \mathcal{B} \\ & w_j \in [0, 1], \quad \forall j \in \mathcal{S} \end{aligned}$$

where $\sigma(c, w) := \frac{1}{N} \left[\underbrace{\sum_{i \in \mathcal{B}} c_i}_{\text{Total bid}}, \underbrace{\sum_{i \in \mathcal{B}} c_i^2}_{\propto R(c, v)}, \underbrace{\sum_{j \in \mathcal{S}} \hat{E}_j w_j}_{\text{Total Energy Sold}} \right]^\top$

Solution Algorithm:

- Gradient-based update with local proxies for global quantities
- Consensus-based update to track global quantities



Algorithm

Notation:

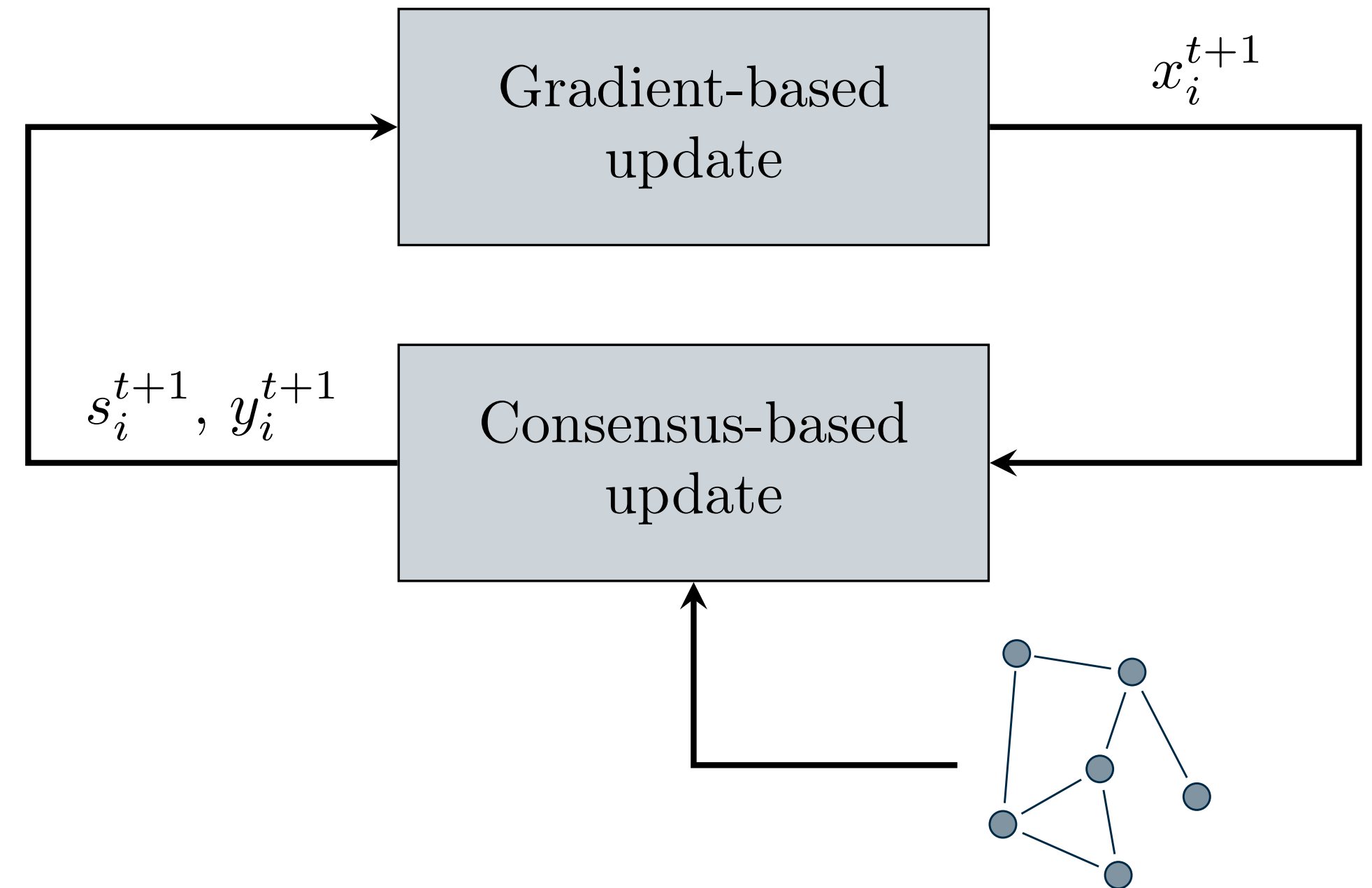
- Buyer: $x_i := c_i$ and $X_i := [c_b, c_s]$
- Seller: $x_i := w_i$ and $X_i := [0,1]$

Gradient based Update:

$$\tilde{x}_i^t = P_{X_i} \left[x_i^t - \gamma \left(\nabla_1 f_i(x_i^t, s_i^t) + \nabla \phi_i(x_i^t) y_i^t \right) \right]$$

Consensus based Update:

$$s_i^{t+1} = \sum_{j \in \mathcal{N}_i} a_{ij} s_j^t + \phi_i(x_i^{t+1}) - \phi_i(x_i^t)$$
$$y_i^{t+1} = \sum_{j \in \mathcal{N}_i} a_{ij} y_j^t + \nabla_2 f_i(x_i^{t+1}, s_i^{t+1}) - \nabla_2 f_i(x_i^t, s_i^t)$$



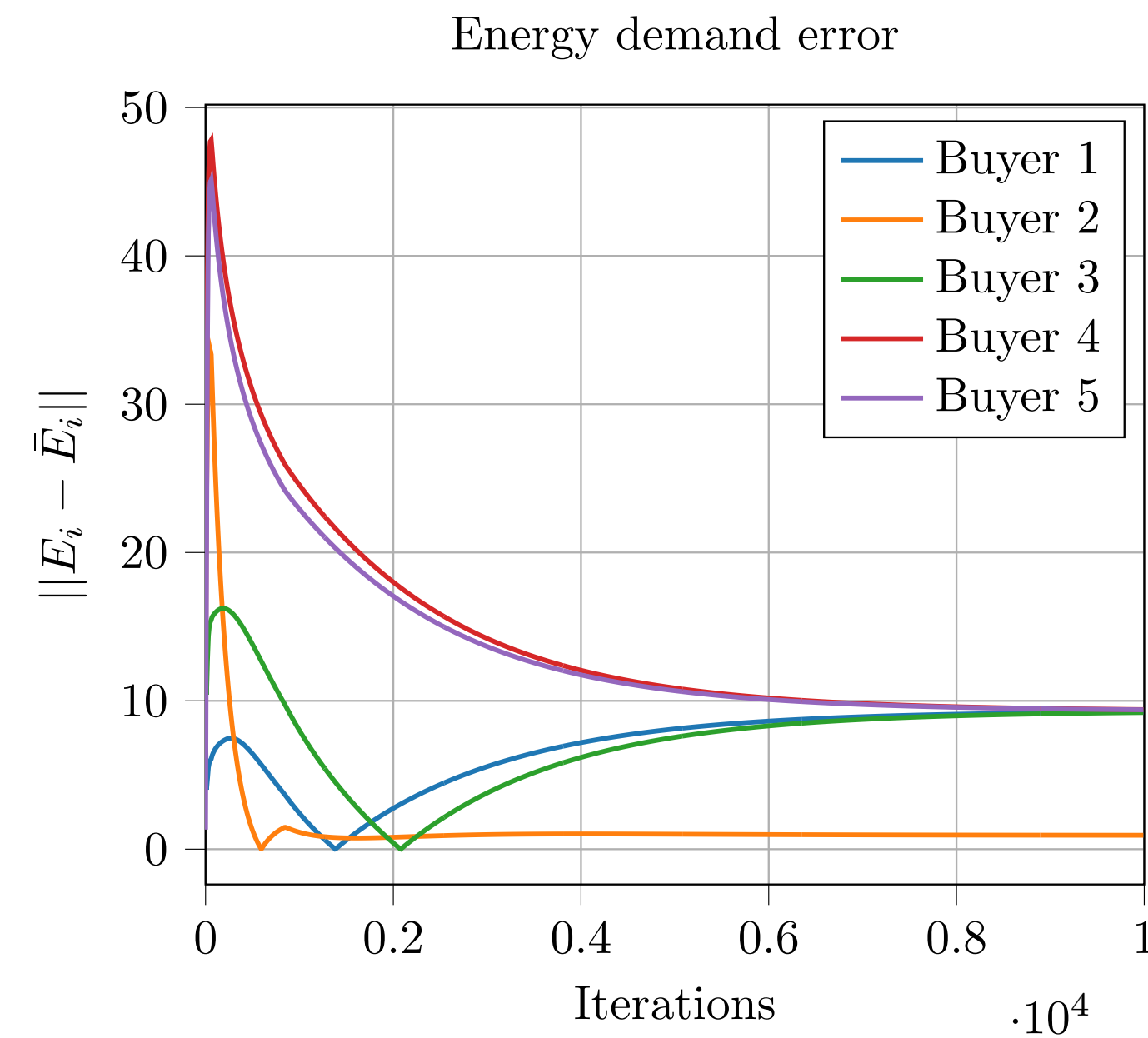
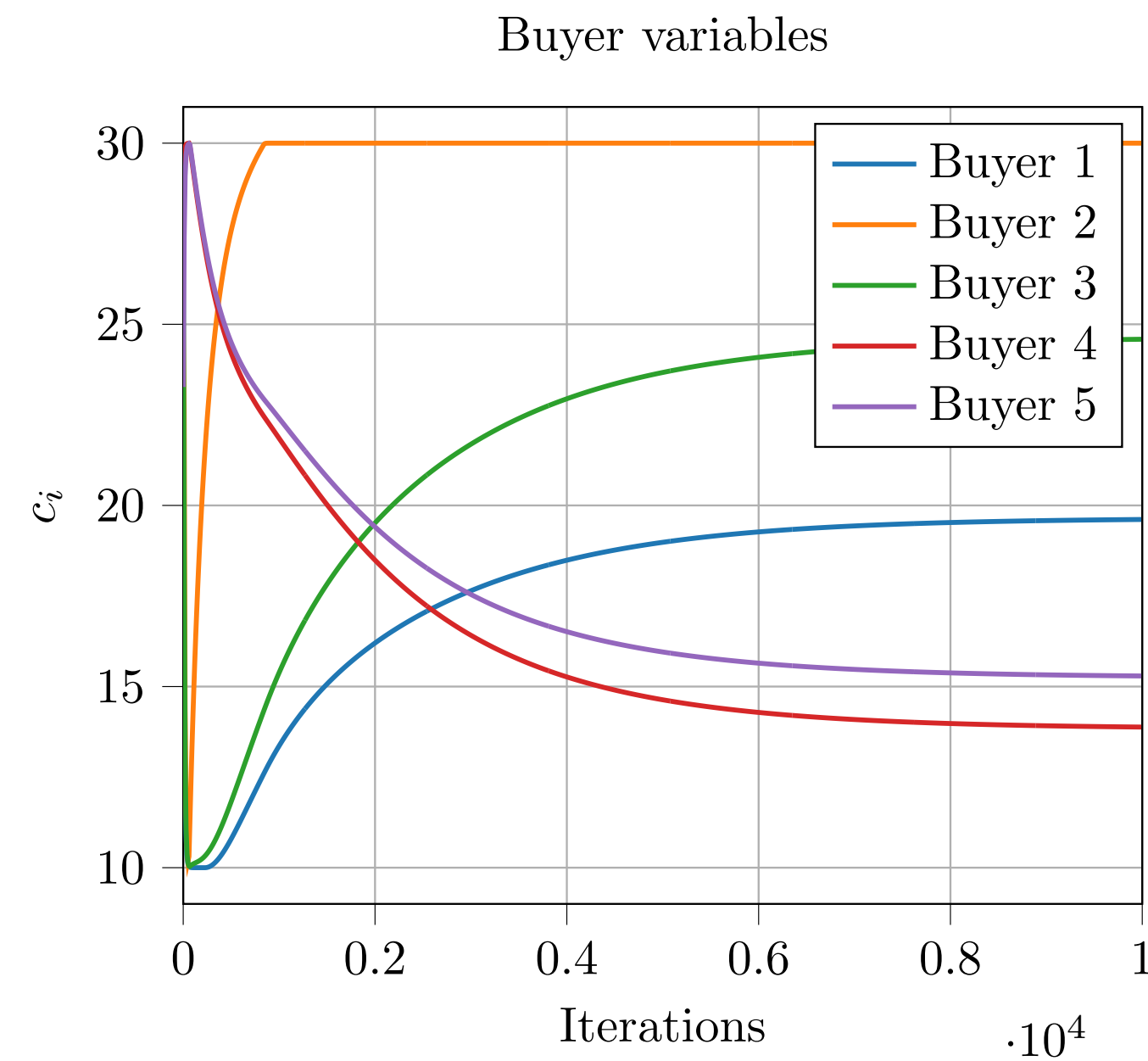
Numerical Simulation: Focus on Demand satisfaction

Desired Behavior:

Buyer 2 prioritizes demand satisfaction

Parameter Selection:

$$\varepsilon_2^d \gg \max \left\{ \varepsilon_i^d, \varepsilon_i^b, \varepsilon_2^b, \gamma_j^r, \gamma_j^s \right\} \quad \forall i \in \mathcal{B} \setminus \{2\}, \forall j \in \mathcal{S}$$



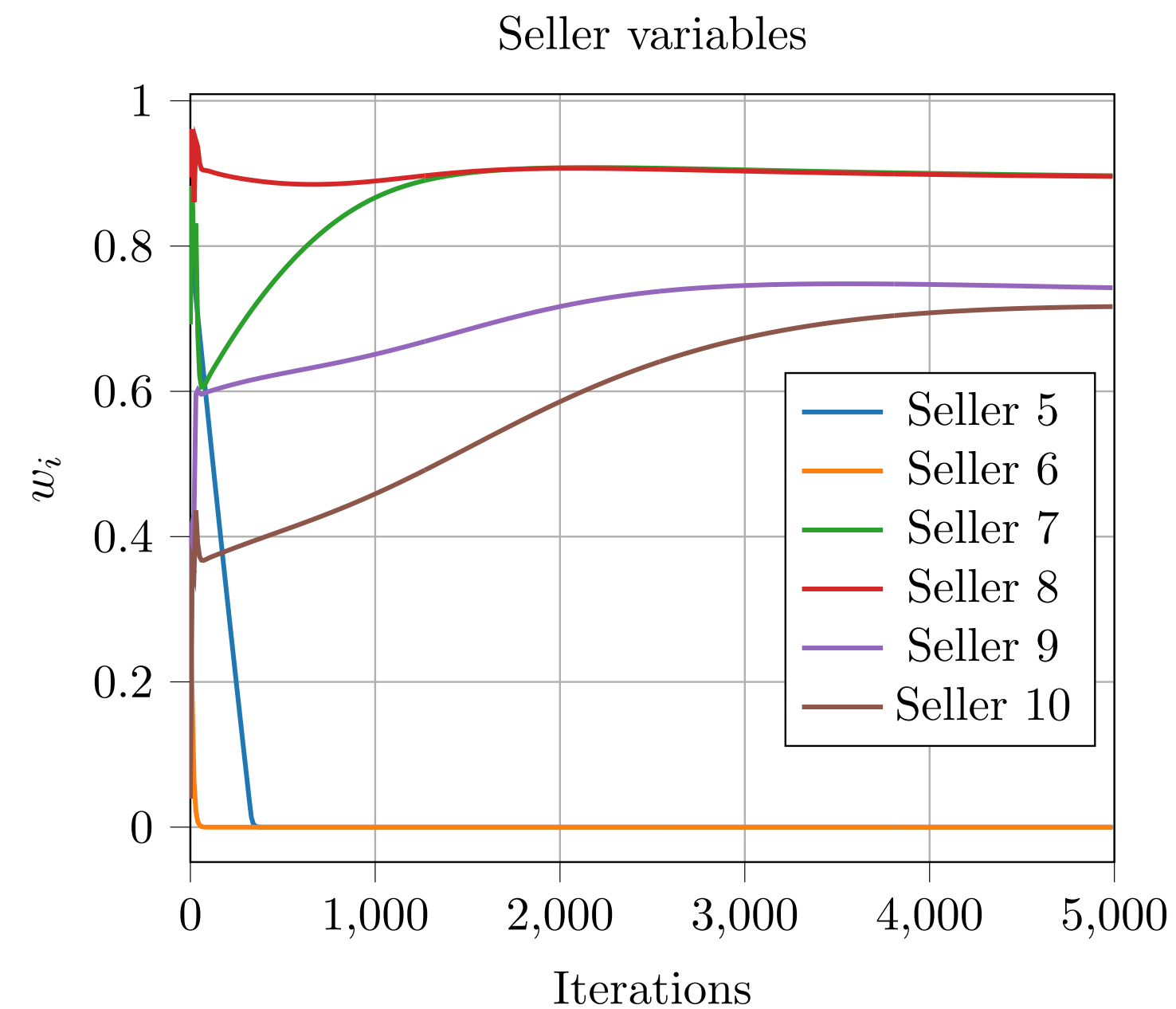
Numerical Simulation: Focus on Storage

Desired Behavior:

Sellers 5 and 6 prioritize energy storage

Parameter Selection:

$$\gamma_5^s, \gamma_6^s \gg \max \left\{ \varepsilon_i^d, \varepsilon_i^b, \gamma_j^r, \gamma_5^r, \gamma_6^r, \gamma_j^s \right\} \quad \forall i \in \mathcal{B}, \forall j \in \mathcal{S} \setminus \{5,6\}$$



Data-Driven Distributed Optimization via Aggregative Tracking and Deep-Learning

Data-driven Distributed Optimization

Distributed setup

- agents communicating over a graph
- agents access only local information

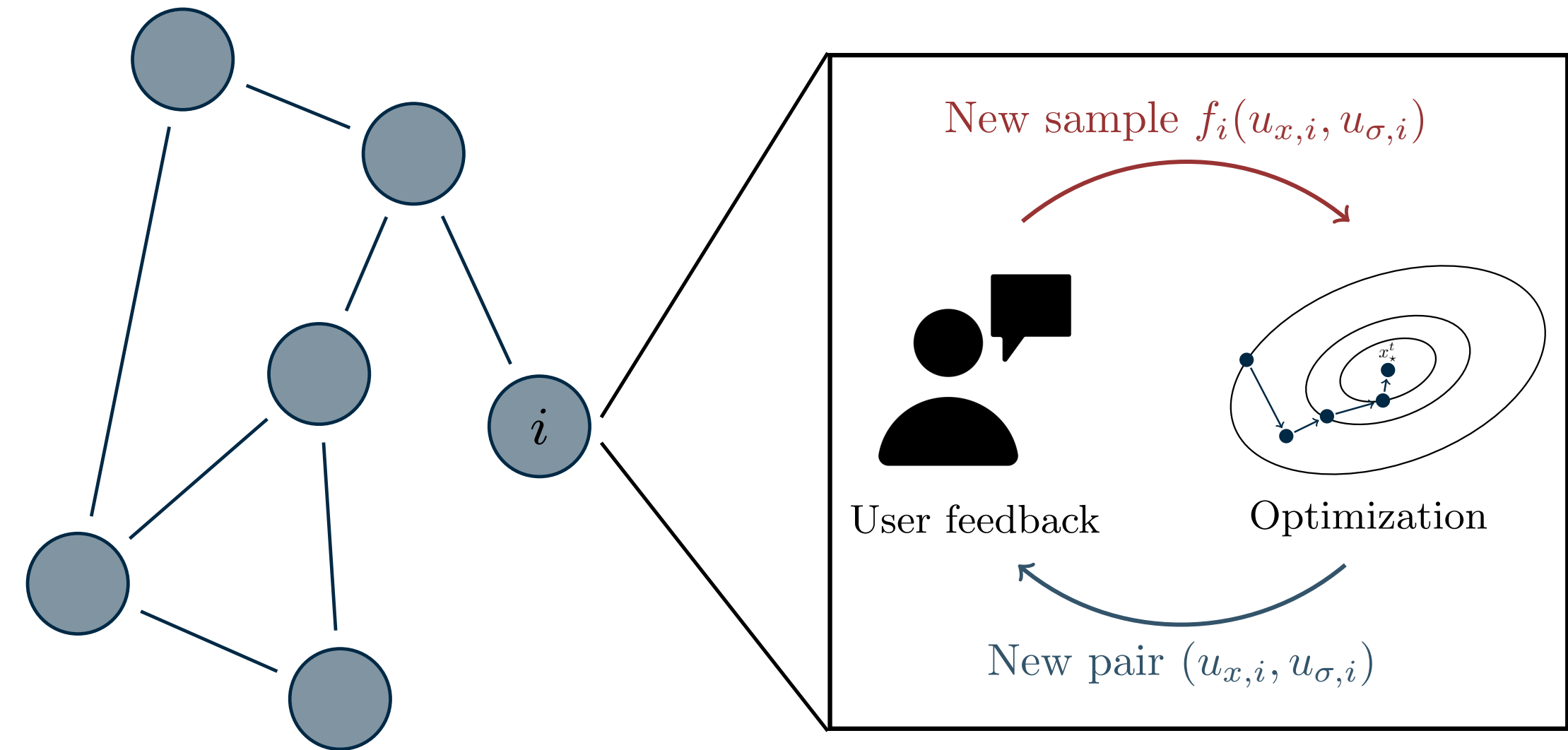
Optimization problem

$$\min_{x_1, \dots, x_N} \sum_{i=1}^N f_i(x_i, \sigma(x))$$

where $\sigma(x) := \frac{1}{N} \sum_{i=1}^N \phi_i(x_i)$ is called aggregative variable

Data-driven scenario

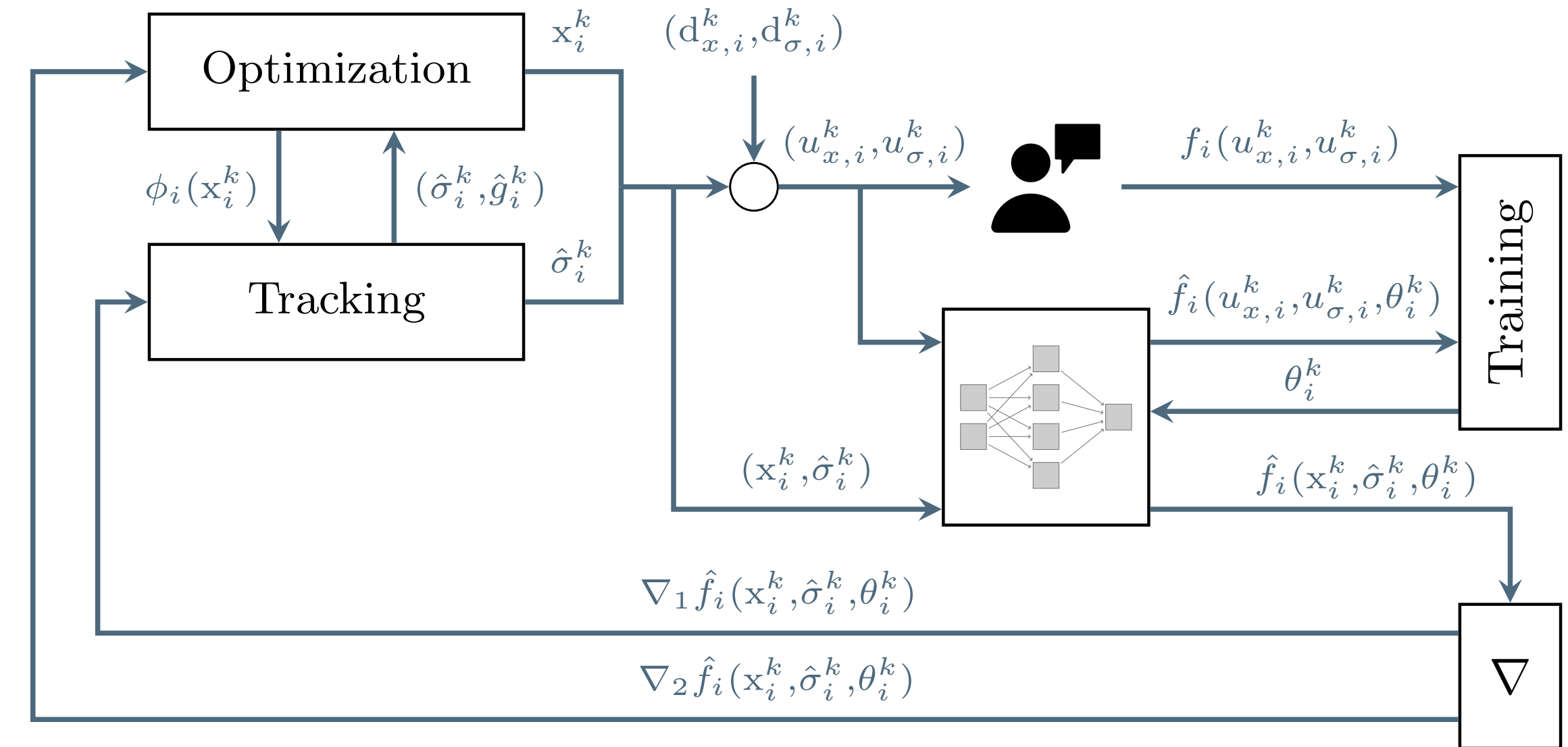
- Unknown local objectives
- One feedback of each i at each iteration



Distributed Strategy and Theoretical Guarantees

Solution Idea:

- Approximate the descent direction as $\nabla \hat{f}_i$
- Learning based on local neural networks
- Update the current solution estimate x_i^k
- Track $\sigma(x)$ and $\frac{1}{N} \sum_{i=1}^N f_i(x_i, \sigma(x))$ via w_i^k and z_i^k



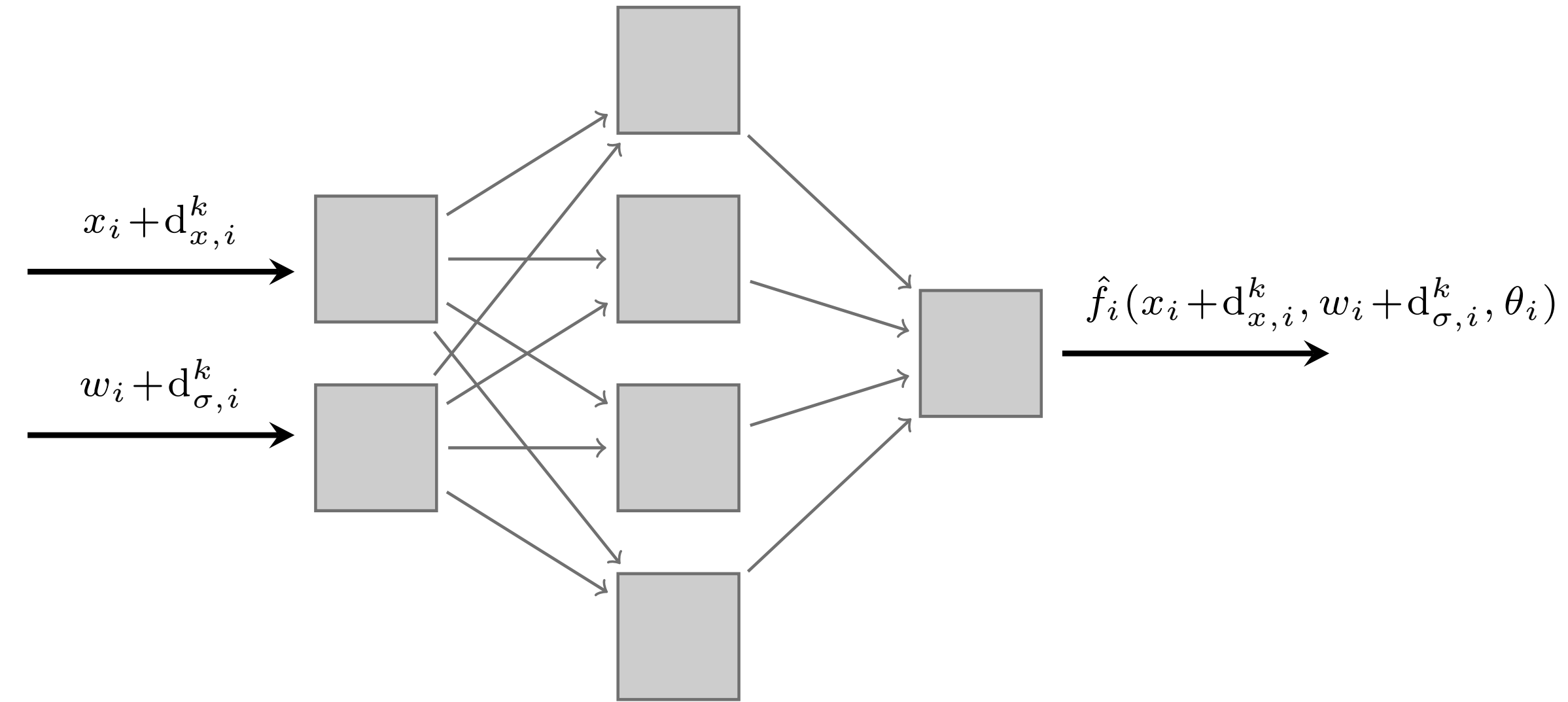
Theoretical Guarantees:

- Convergence to a neighborhood of the optimum
- Proof based on timescale separation and averaging theory

Learning Strategy

Idea: for each iteration k

- Given a neural network with parameter $\theta_i \in \mathbb{R}^{m_i}$
- Sample f_i in a point around the current $(x_i, \sigma(x))$
- Learn $f_i(\cdot, \cdot)$ from samples as $\hat{f}_i(\cdot, \cdot, \theta)$



Training cost function:

$$\ell_i(x_i^k, w_i, \theta_i) := \frac{1}{2} \left\| f_i\left(x_i^k + d_k^{x,i}, w_i + \hat{d}_k^{\sigma,i}\right) - \hat{f}_i\left(x_i^k + d_k^{x,i}, w_i + d_k^{\sigma,i}, \theta_i\right) \right\|^2$$

Gradient estimate: $\nabla f_i(\cdot, \cdot) \approx \nabla \hat{f}_i(\cdot, \cdot, \theta)$

$$\nabla_1 f_i(x_i, w_i, \theta_i) \text{ proxy for } \nabla_1 f_i(x_i, w_i), \quad \nabla_2 f_i(x_i, w_i, \theta_i) \text{ proxy for } \nabla_2 f_i(x_i, w_i)$$

Optimization and Tracking Steps

Idea: for each iteration k

- Follow an approximated gradient method
- Track global quantities with local proxies $w_i^k, z_i^k \in \mathbb{R}^d$

Optimization Step:

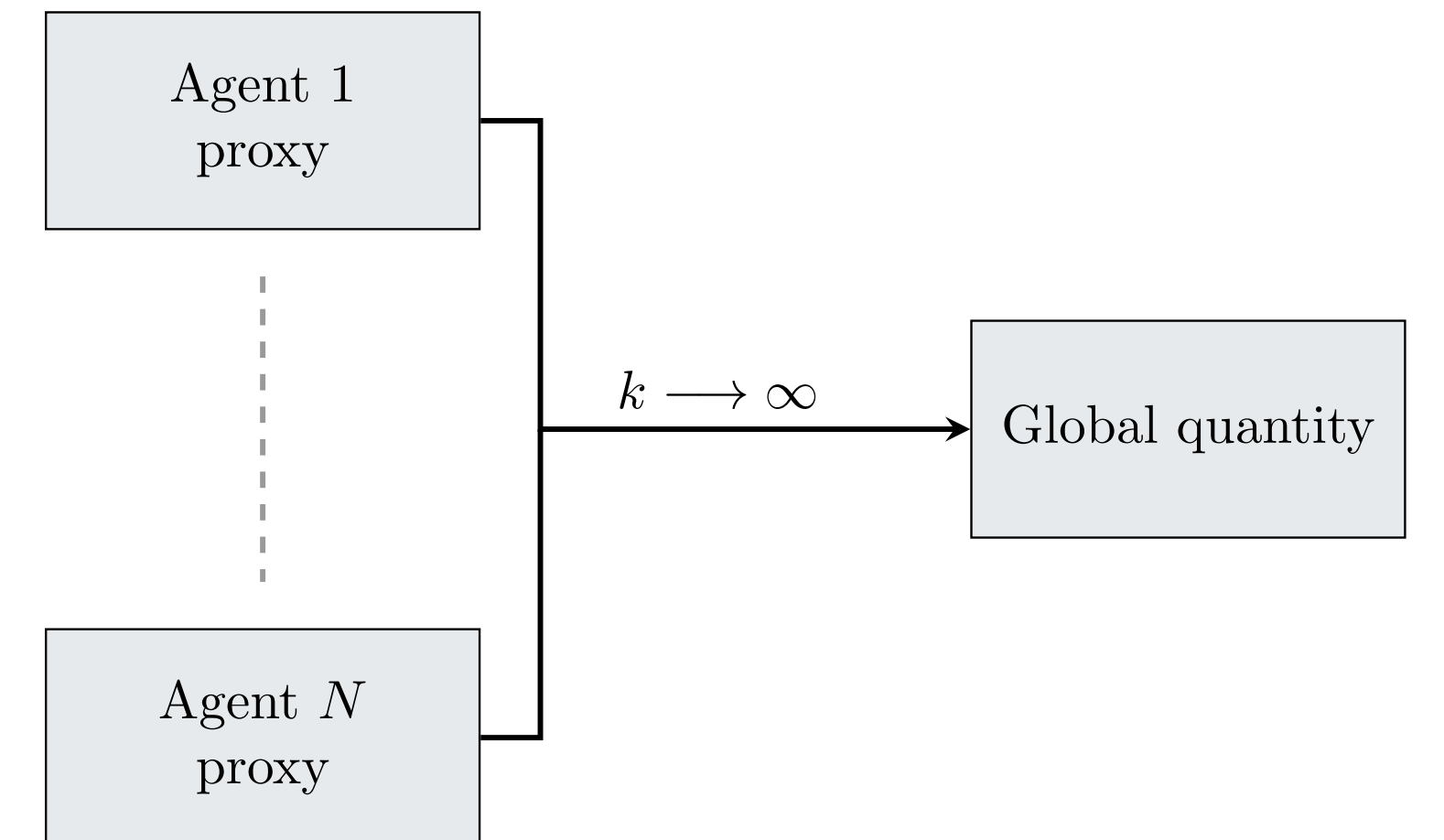
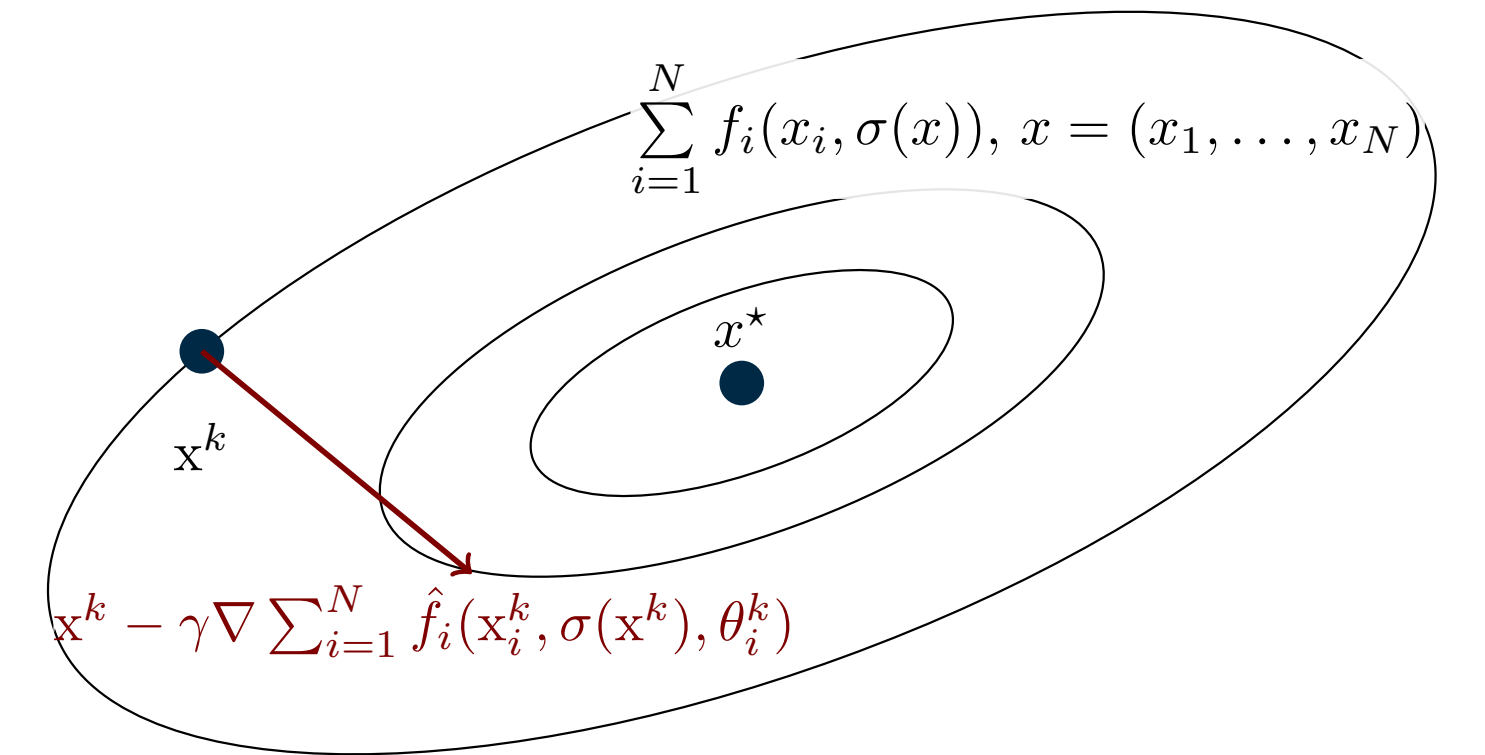
$$x_i^{k+1} = x_i^k - \gamma \nabla_{x_i} \hat{f}_i(x_i^k, w_i^k + \phi_i(x_i^k), \theta_i^k)$$

Tracking Step:

Consensus based update such that

$$w_i^k + \phi_i(x_i^k) \xrightarrow[k \rightarrow \infty]{} \sigma(x^k)$$

$$z_i^k + \nabla_2 \hat{f}_i(x_i^k, w_i^k + \phi_i(x_i^k), \theta_i^k) \xrightarrow[k \rightarrow \infty]{} \frac{1}{N} \sum_{j=1}^N \nabla_2 \hat{f}_j(x_j^k, \sigma(x^k), \theta_j^k)$$



Brumali, Carnevale, Notarstefano, "Data-driven Distributed Optimization via Aggregative Tracking and Deep Learning", in 2025 submitted to journal (conf. L4DC 2024)

DEep-Learning aggregative TrAcking (DELTA)

Learning update

$$\theta_i^{k+1} = \theta_i^k - \gamma \nabla_3 \ell_i \left(x_i^k + d_{x,i}^k, w_i^k + \phi_i(x_i^k) + d_{\sigma,i}^k, \theta_i^k \right)$$

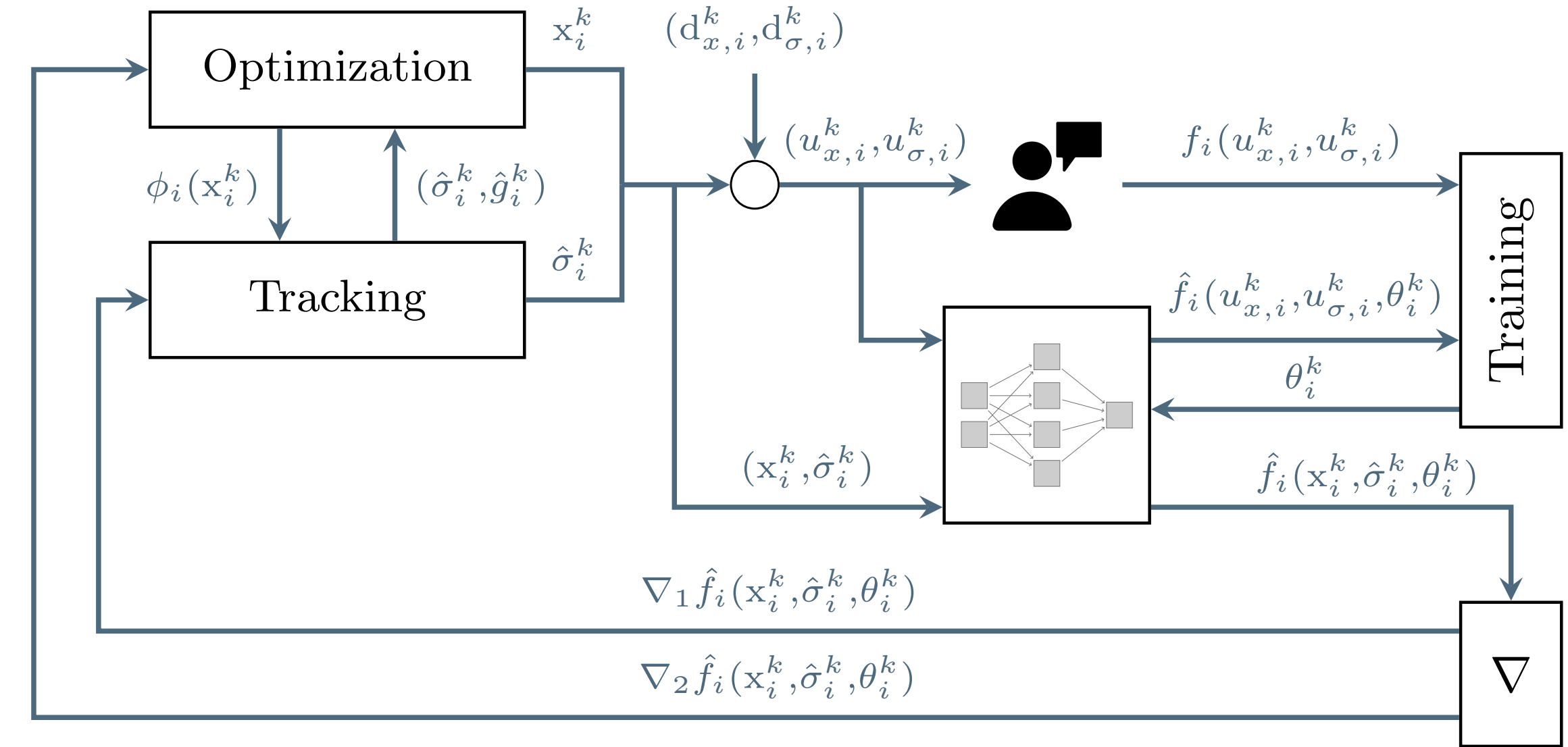
Optimization update

$$x_i^{k+1} = x_i^k - \gamma \left[\nabla_1 \hat{f}_i \left(x_i^k, w_i^k + \phi_i(x_i^k), \theta_i^k \right) + \nabla \phi_i(x_i^k) \left(z_i^k + \nabla^2 \hat{f}_i \left(x_i^k, w_i^k + \phi_i(x_i^k), \theta_i^k \right) \right) \right]$$

Tracking update

$$w_i^{k+1} = \sum_{j \in \mathcal{N}_i} a_{ij} \left(w_j^k + \phi_j(x_j^k) \right) - \phi_i(x_i^k),$$

$$z_i^{k+1} = \sum_{j \in \mathcal{N}_i} a_{ij} \left(z_j^k + \nabla_2 \hat{f}_j \left(x_j^k, w_j^k + \phi_j(x_j^k), \theta_j^k \right) \right) - \nabla_2 \hat{f}_i \left(x_i^k, w_i^k + \phi_i(x_i^k), \theta_i^k \right).$$



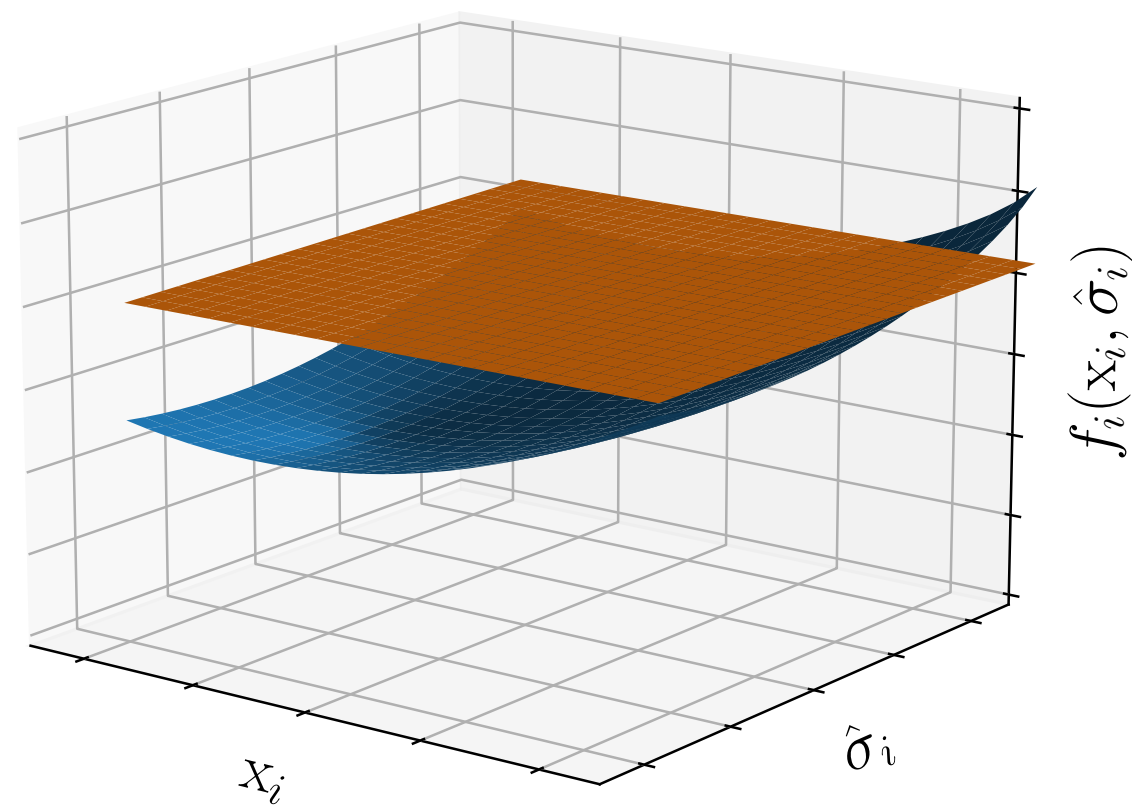
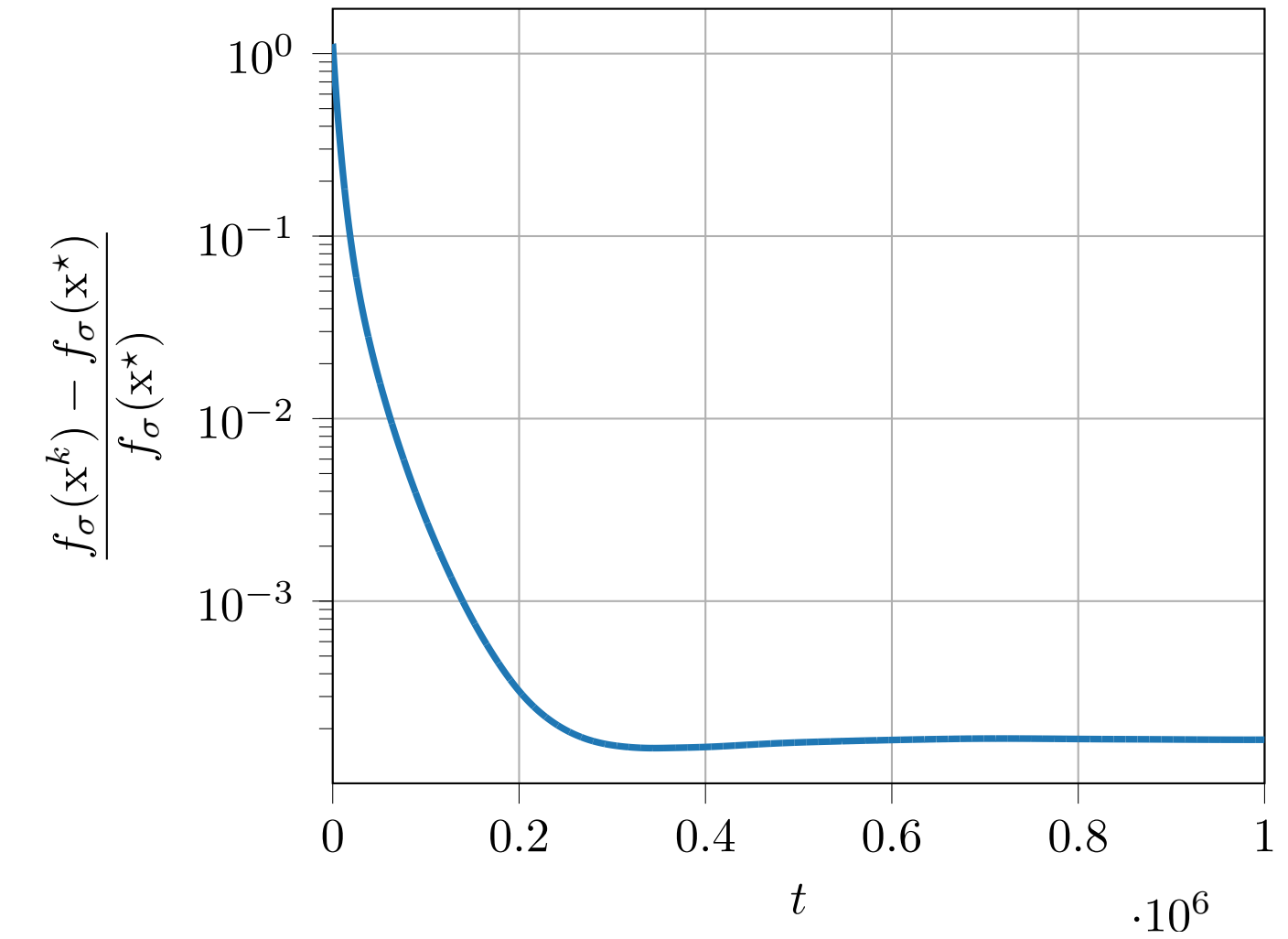
Numerical Simulations

Setup:

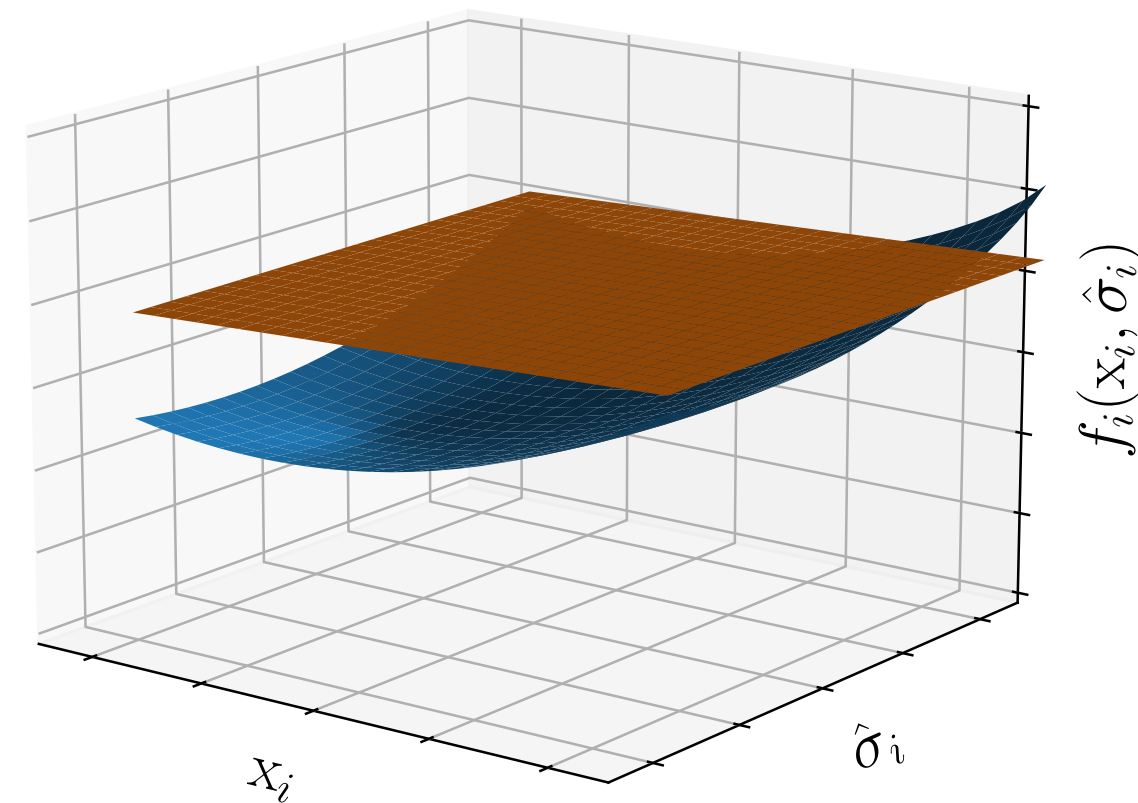
- $N = 20$ agents
- Networks structure: 2 layer 300 neurons

Local f_i :

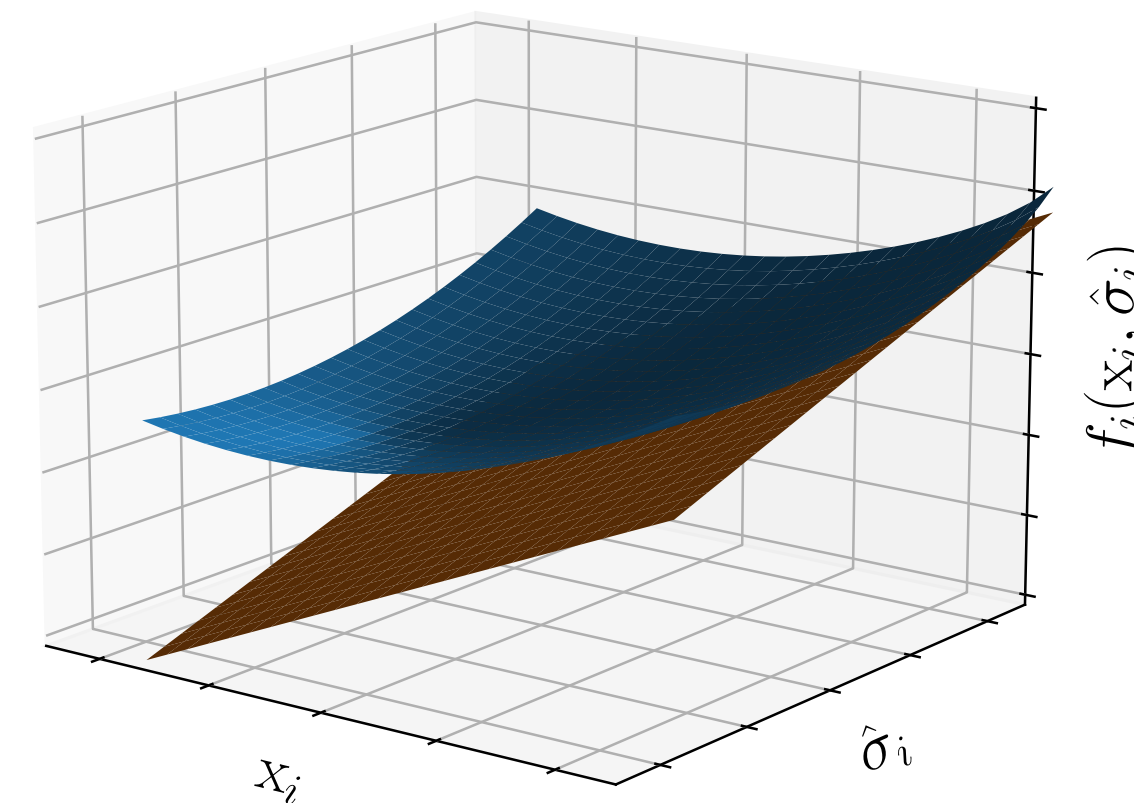
$$f_i(x_i, \sigma(x)) = \frac{1}{2} \left[\begin{bmatrix} x_i \\ \sigma(x) \end{bmatrix}^\top P_i \begin{bmatrix} x_i \\ \sigma(x) \end{bmatrix} + v_i^\top \begin{bmatrix} x_i \\ \sigma(x) \end{bmatrix} + a_i + e^{-b_i} \begin{bmatrix} x_i \\ \sigma(x) \end{bmatrix}^{+c_i} + q_i \right]$$



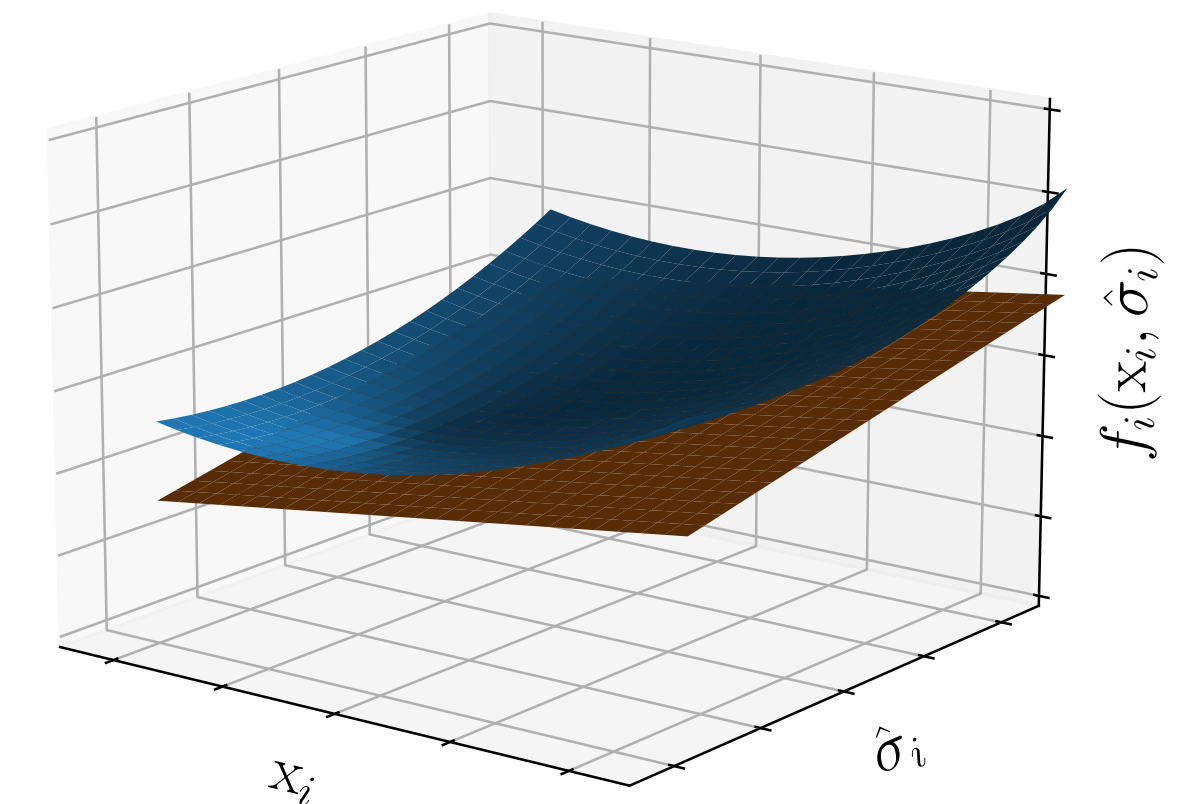
$k = 0$



$k = 20$



$k = 500$



$k = 10^4$

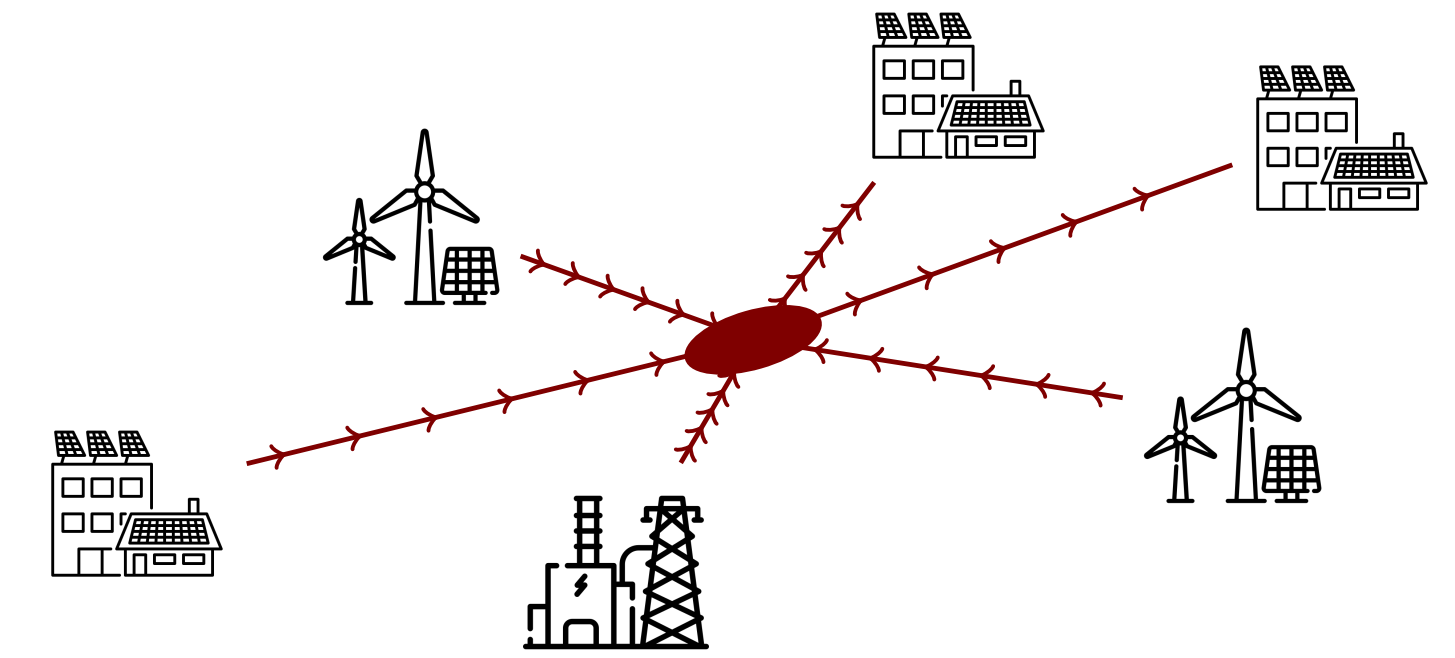
Brumali, Carnevale, Notarstefano, "Data-driven Distributed Optimization via Aggregative Tracking and Deep Learning", in 2025 submitted to journal (conf. L4DC 2024)

Distributed Learning and Optimization
of a Multi-Agent Macroscopic Probabilistic Model

Distributed Learning and Optimization of a Multi-Agent Macroscopic Probabilistic Model

Multi-agents systems have been widely studied in optimization, e.g.,

- Energy communities coordination
- Multi-robot applications



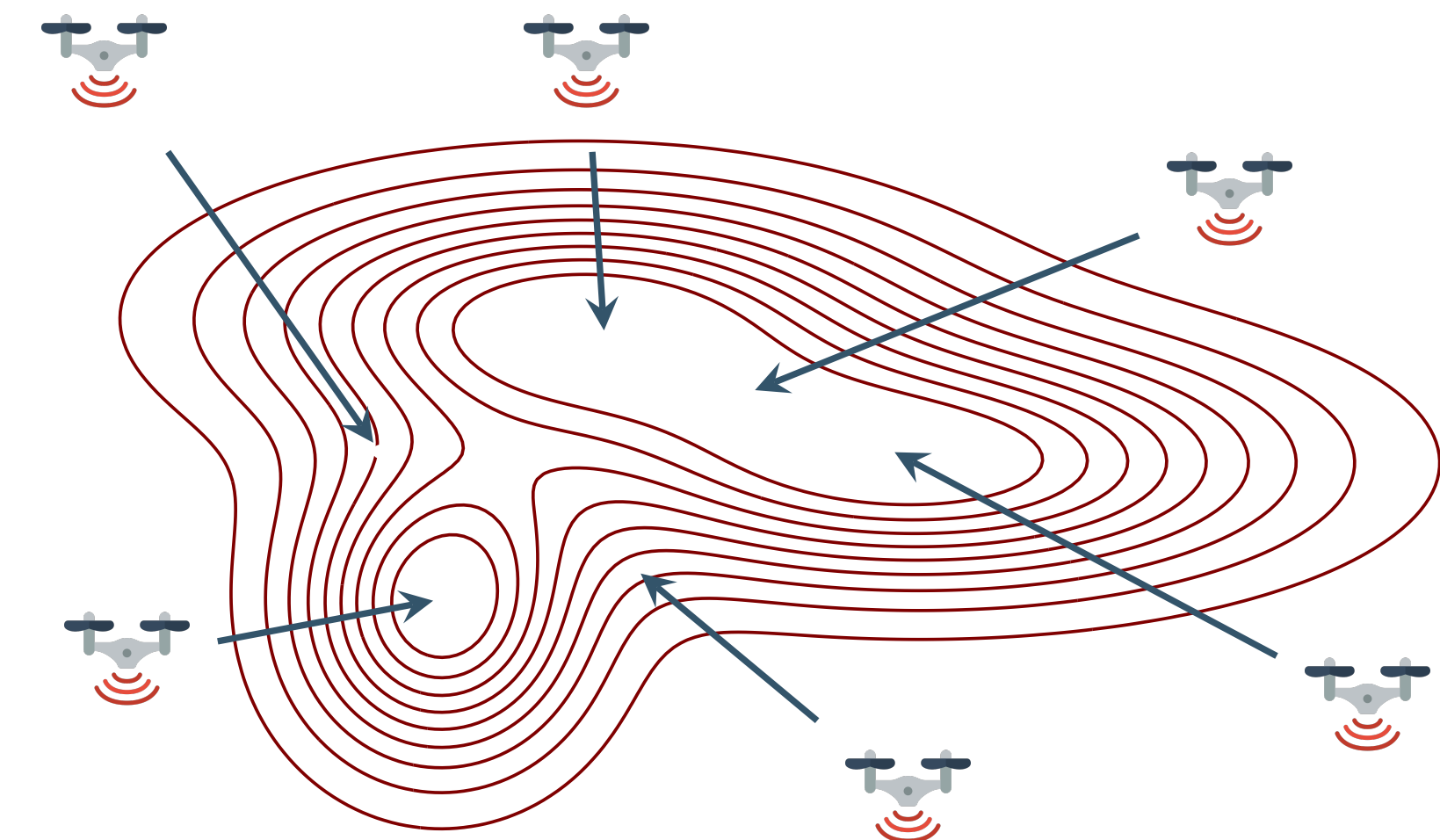
Agents can coordinate themselves to make a certain behavior emerge

Existing works based on two approaches to coordinate agents

- Centralized approach to optimize spatial distribution (based on PDEs)
- Distributed algorithms to coordinate single agents

Goal:

- Address the optimization in one step at a multi-scale level
- Act directly on the single agents to optimize the behavior of the ensemble



Problem Setup

Setup:

- N interacting agents modeled as a continuum
- System described by a macroscopic probabilistic model $p(x)$
- Discretized workspace into C cells

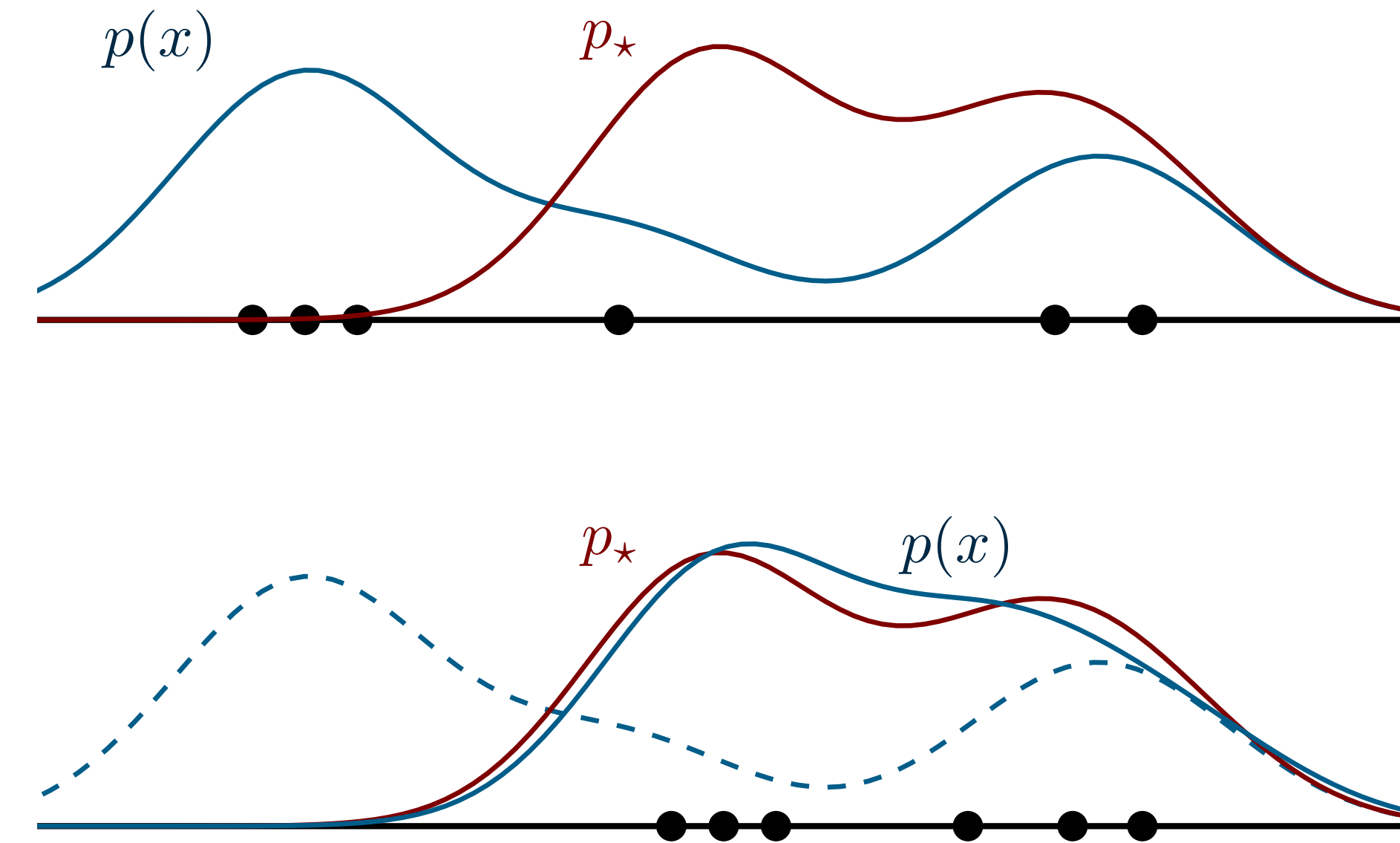
Goal:

Act on microscopic states x_i to achieve a macroscopic behavior p_\star

Optimization Problem:

$$\min_{x_1, \dots, x_N} \sum_{c=1}^C \ell(p^c(x))$$

with $p^c(x)$ agents macroscopic model at cell c , $\ell(p^c(x))$ performance index (Kullback-Leibler divergence)

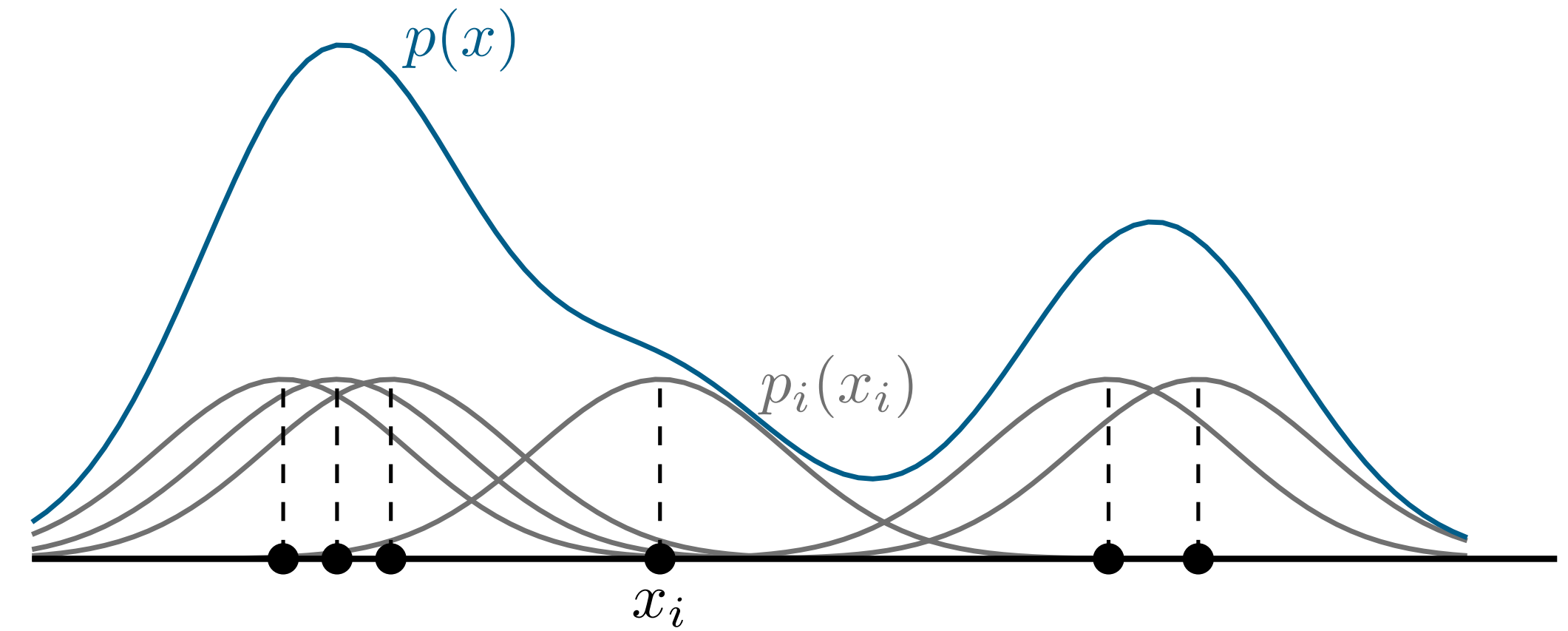


Solution Strategy

Assumption: for each cell c

- Agents possess local models $p_i^c(x_i)$
- Kernel Density Estimation for macroscopic model $p^c(x)$

Macroscopic probabilistic model: $p^c(x) = \frac{1}{N} \sum_{i=1}^N p_i^c(x_i)$



Algorithm for agent i :

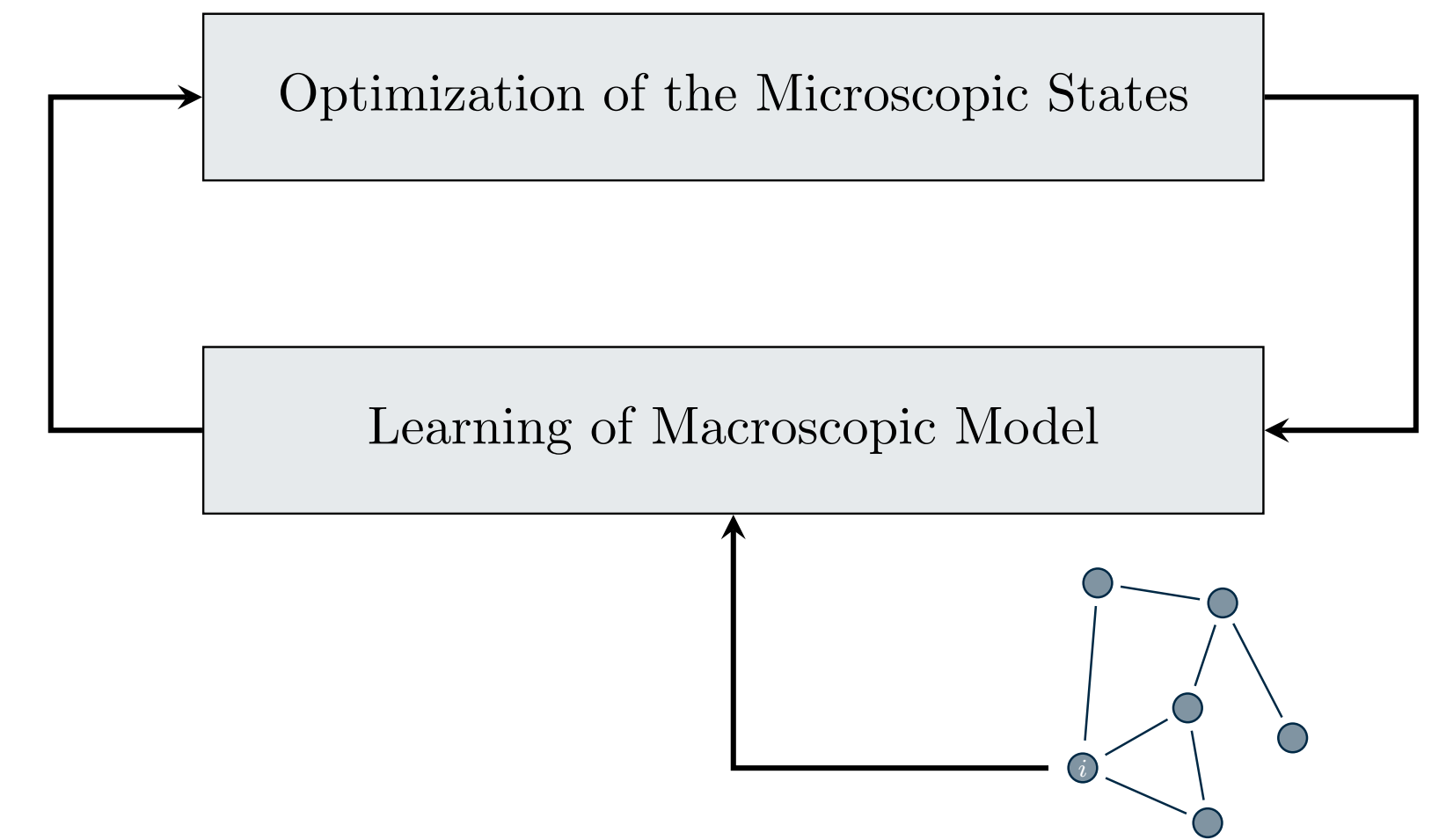
- Microscopic state update:

$$x_i^{k+1} = x_i^k - \gamma \frac{1}{N} \nabla p_i(x_i^k) \frac{d}{dp} \ell(p) \Big|_{p=P_{\mathcal{E}}(w_i^k + p_i(x_i^k))}$$

- Macroscopic model learning:

$$w_i^{k+1} = \sum_{j \in \mathcal{N}_i} a_{ij} w_j^k + p_j(x_j^k) - p_i(x_i^k)$$

Microscopic Perspective (Agent i)

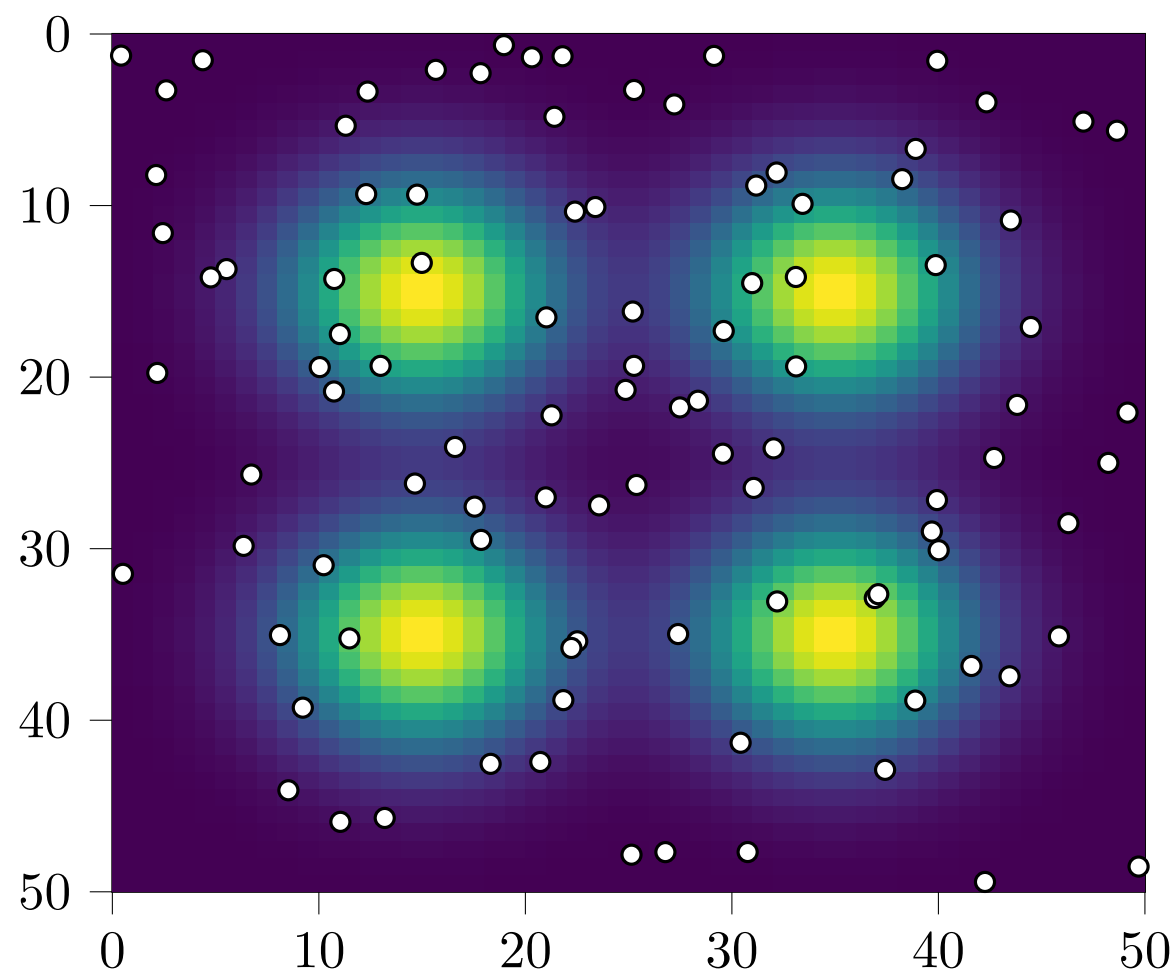
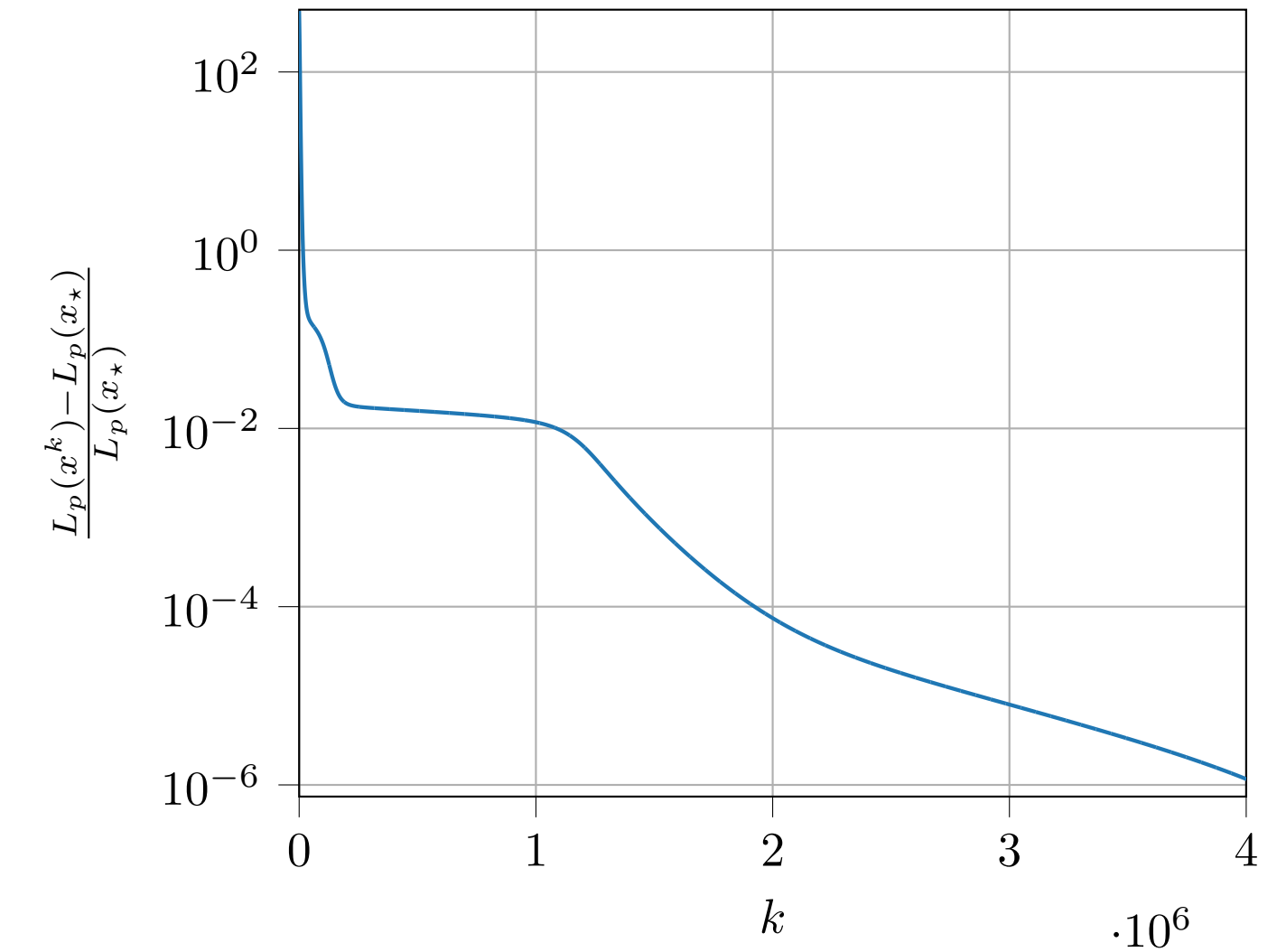


Numerical Simulation

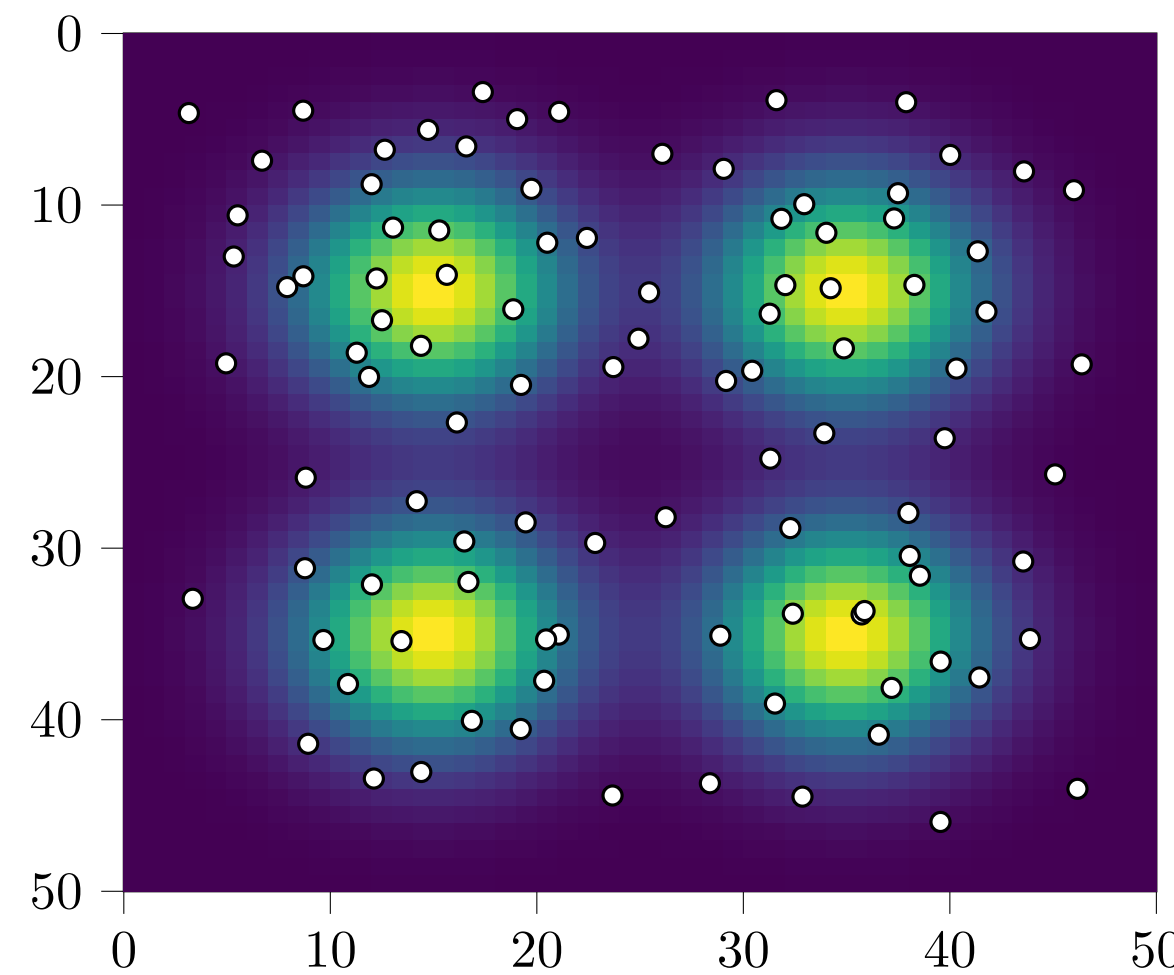
Scenario: Sensor network deployment problem for event detection

Simulation setup:

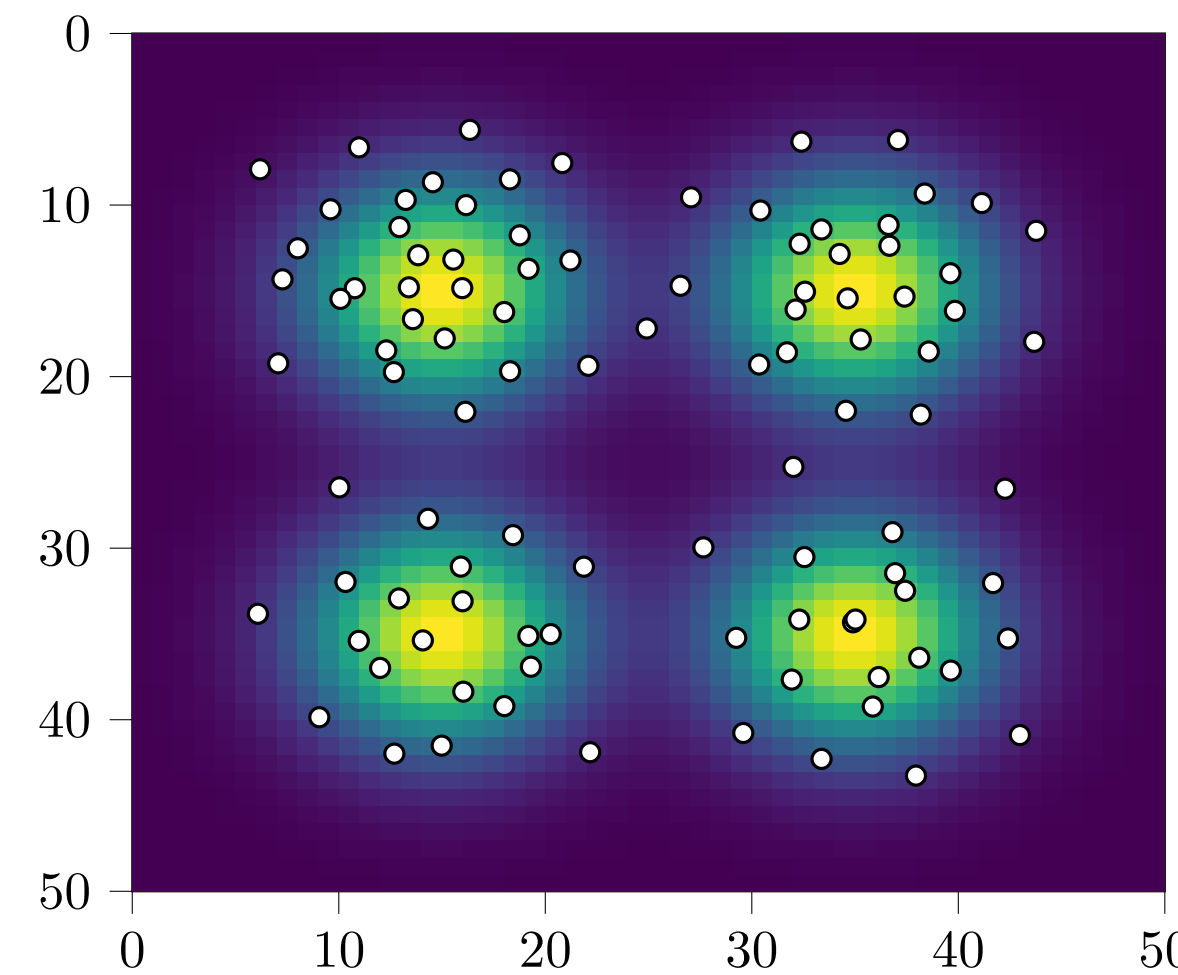
- Space $C := [0,100] \times [0,100]$ discretized with $C = 2500$
- Event PDF p_\star mixture of 4 Gaussians
- $N = 100$ agents with Gaussian $p_i(x_i)$



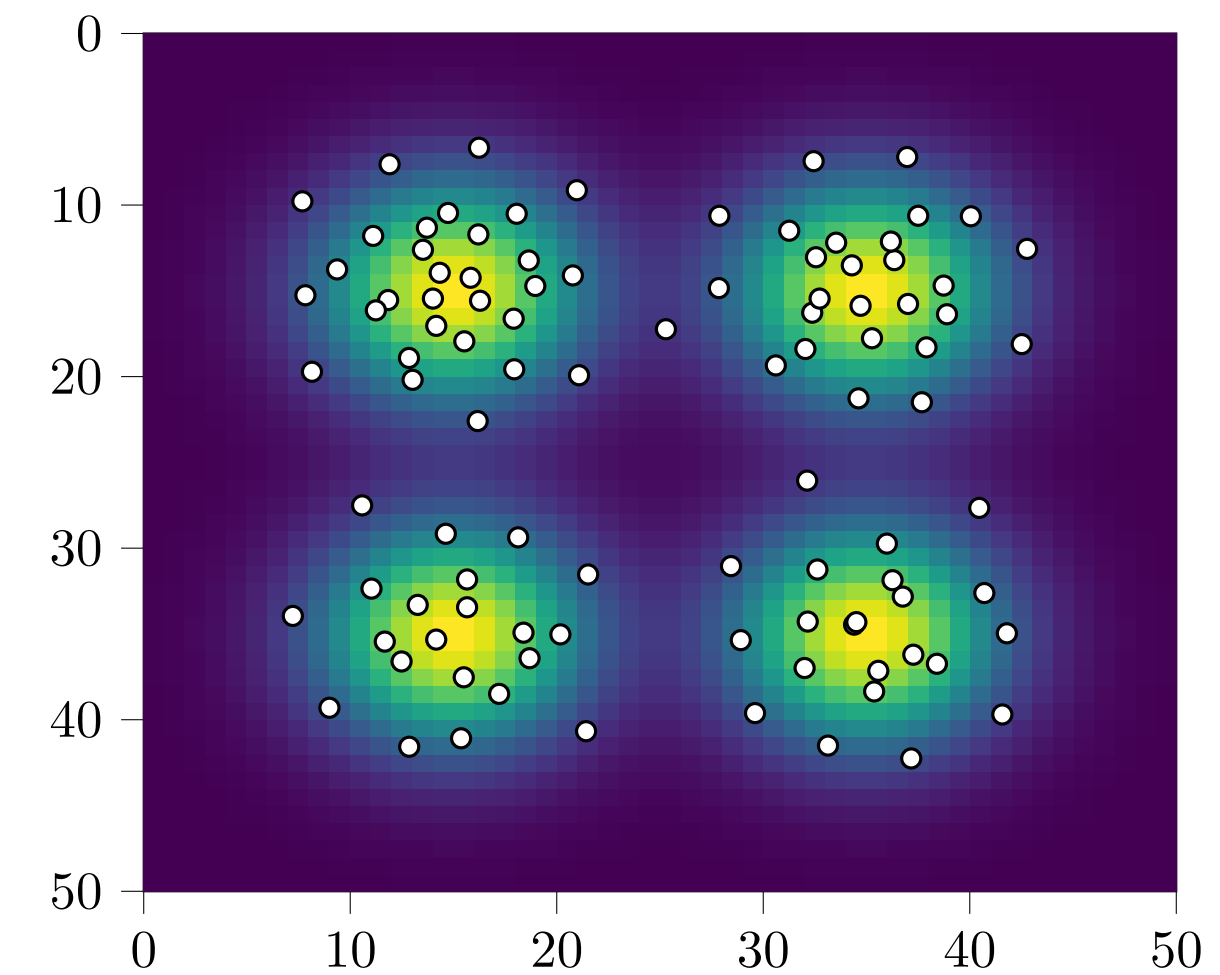
$k = 0$



$k = 20000$



$k = 50000$



$k = 100000$

Brumali, Carnevale, Notarstefano (2025). Distributed learning and optimization of a multi-agent macroscopic probabilistic model. European Journal of Control, 101332.

Application of Distributed Optimization to the management
of active and reactive powers in sustainable microgrids

Joint work with UNIGE

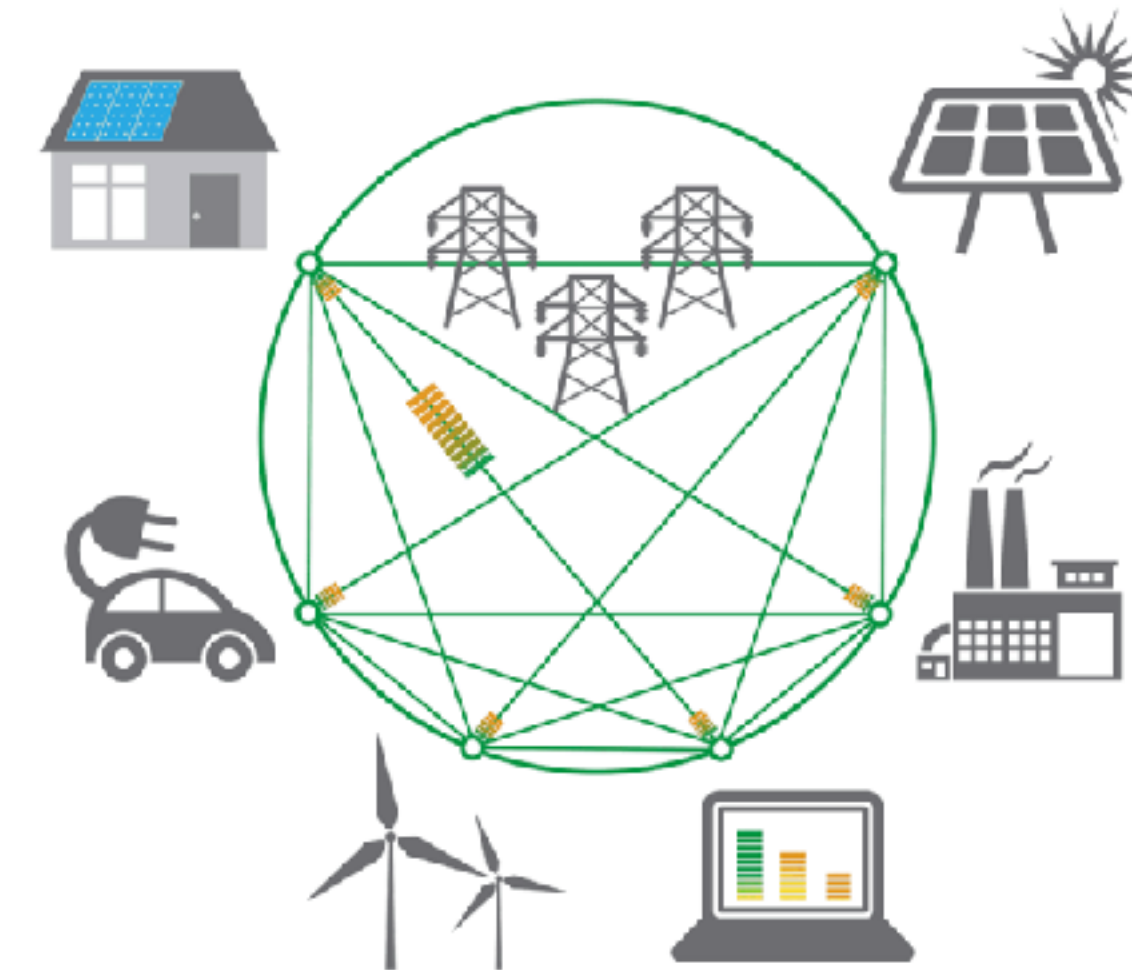
Application Setting

Consider a **microgrid** system with N nodes:

- H controllable microturbines
- L renewable sources
- K storage systems
- Main grid
- Boiler

Distributed Setup:

- Each node is a different agent
- Only local information available
- Graph based communication



Goal: Design and implement a distributed strategy to optimize power production satisfying all constraints

Joint work with the University of Genova “Distributed Optimization Algorithm for the management of active and reactive powers in sustainable microgrids”

Agent Model: Microturbine

Decision Variables:

- $p_{h,t}^{el} \in [p_{h,t}^{el}, \bar{p}_{h,t}^{el}]$ electrical power produced at time t
- $F_{h,t}^{utx}$, $F_{h,t}^{tx}$ untaxed and taxed fuel used by boilers at time t

Constraints: at time t

- $F_{h,t}^{utx} = (1 - ff) \beta p_{h,t}^{el}$ with ff and β taxation parameters
- $F_{h,t} = F_{h,t}^{utx} + F_{h,t}^{tx}$ with $F_{h,t}$ total fuel used by boilers
- $p_{h,t}^{pe} = F_{h,t} L_{hv}$ with $p_{h,t}^{pe}$ primary energy for the microturbines

Cost Function:

$$J(p_{h,t}^{pe}, F_{h,t}^{utx}, F_{h,t}^{tx}) = \Delta \sum_{t=0}^{T-1} \left(F_{h,t}^{utx} K_{utx} + F_{h,t}^{tx} K_{tx} + C_t p_{h,t}^{pe} \right)$$

Where Δ interval length, K_{utx} , K_{tx} and C_t fuels and emissions cost

Agent Model: Renewable source

Each renewable produce $\bar{p}_{l,t}^{res}$ of active power at time t

Decision Variables:

- $p_{l,t}^{res}$ active power exploited at time t
- $q_{l,t}^{res}$ reactive power produced at time t

Constraints:

Use at most the produced power

- $p_{l,t}^{res} \in [0, \bar{p}_{l,t}^{res}]$
- $q_{l,t}^{res} \in [\underline{q}_{l,t}^{res}, \bar{q}_{l,t}^{res}]$

Agent Model: Storage Unit

Decision Variables:

- $p_{k,t}^s \in [\underline{p}_{k,t}^s, \bar{p}_{k,t}^s]$ active power exchanged with storages at time t
- $q_{k,t}^s \in [\underline{q}_{k,t}^s, \bar{q}_{k,t}^s]$ reactive power exchanged with storages at time t

Constraint: SoC e_k constraints

- $e_{k,t+1} = a_k e_{k,t} + \eta_k p_{k,t}^s \Delta$
- $e_{k,t} \in [\underline{e}_{k,t}, \bar{e}_{k,t}]$

Where $\eta_k \in [0,1]$ is the charging/discharging efficiency $a_k \in [0,1]$ a decay factor

Agent Model: Main Grid

Decision Variables:

- $p_t^{\text{grid}}, q_t^{\text{grid}}$ Active and reactive power exchanged between units and grid
- $p_t^{\text{grid},i}, p_t^{\text{grid},o}$ Power bought and sold with the main grid

Constraint:

- $p_t^{\text{grid},i} - p_t^{\text{grid},o} = p_t^{\text{grid}}$
- $p_t^{\text{grid},i}, p_t^{\text{grid},o} \geq 0$
- $p_t^{\text{grid}} \in [p_t^{\text{grid}}, \bar{p}_t^{\text{grid}}]$ and $q_t^{\text{grid}} \in [q_t^{\text{grid}}, \bar{q}_t^{\text{grid}}]$

Cost Function:

$$J(p_t^{\text{grid},i}, p_t^{\text{grid},o}) = \Delta \sum_{t=0}^{T-1} (B p_t^{\text{grid},i} - S p_t^{\text{grid},o})$$

With B and S the buying and selling prices respectively

Agent Model: Boiler

Decision Variables:

- $p_t^{pe,b}$ Primary energy fed to the boiler
- $p_t^{th,b}$ Thermal power produced by the boiler

Constraint:

$$p_t^{pe,b} \eta_B = p_t^{th,b} \quad \text{with } \eta_B \in [0,1] \text{ the boiler efficiency}$$

Cost Function:

$$J(p_t^{pe,b}, p_t^{th,b}) = \Delta \sum_{t=0}^{T-1} \left(E p_t^{th,b} + C p_t^{pe,b} \right)$$

Electrical and Thermal Balance

The system must satisfy these constraints depending on all agents variables

Active and Reactive Power Balance:

$$\sum_{h \in H} p_{h,t}^{el} + \sum_{l \in L} p_{l,t}^{res} - \sum_{k \in K} p_{k,t}^s + p_t^{grid} = p_t^d + p_t^{veh}$$

$$\sum_{l \in L} q_{l,t}^{res} - \sum_{k \in K} q_{k,t}^s + q_t^{grid} = q_t^d + q_t^{veh}$$

Where p_t^d , q_t^d , p_t^{veh} , q_t^{veh} are the active and reactive power exchanged with the load

Thermal Power Balance:

$$\sum_{h \in H} p_{h,t}^{th} + p_t^{th,b} + \sum_{l \in L} p_{l,t}^{res} \in [\bar{D}^t, \underline{D}^t]$$

With $p_{h,t}^{th} = \bar{\mu}_h p_{h,t}^{el}$, where $\bar{\mu}_h$ is a conversion parameter

Distributed Optimization Problem

Notation:

- $x_i \in X_i$ decision variables of unit i with associated cost $J_i(x_i)$
- $h_i(x_i)$ and $g_i(x_i)$ influence of x_i in the electric and thermal balance
- $b := \text{col}(p_t^d + p_t^{veh}, q_t^d + q_t^{veh})$ and $\kappa := \text{col}(-\underline{D}^t, \bar{D}^t)$

Constrained Coupled Optimization Problem:

$$\begin{aligned} \min_{x_1, \dots, x_N} \quad & \sum_{i=1}^N J_i(x_i) \\ \text{s.t.} \quad & x_i \in X_i, \quad i = 1, \dots, N, \\ & \sum_{i=1}^N h_i(x_i) = b, \quad \sum_{i=1}^N g_i(x_i) \leq \kappa \end{aligned}$$

Solution Strategy:

Distributed Primal Decomposition

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Solution Algorithm:

Idea:

- recast the problem into a master-subproblem architecture
- See b and κ as shared resources and introduce allocation vectors $y_{\text{eq},i}$ and $y_{\text{in},i}$

Initialization: $y_{\text{eq},i}^0, y_{\text{in},i}^0 : \sum_{i=1}^N y_{\text{eq},i}^0 = \kappa, \quad \sum_{i=1}^N y_{\text{in},i}^0 = b, \quad M > 0$ sufficiently large.

For $k = 0, 1, \dots$

Step 1: Compute $(x_i, \rho_i, \mu_i, \lambda_i)$ solving

$$\begin{aligned} \min_{x_i \in X_i, \rho_i \geq 0} \quad & f_i(x_i) + M\rho_i \\ \text{s.t. } \mu_i : \quad & g_i(x_i) \leq y_{\text{in},i}^k + \rho_i \mathbf{1}_s, \\ \lambda_i : \quad & |h_i(x_i) - y_{\text{eq},i}^k| \leq \rho_i \mathbf{1}_p, \end{aligned}$$

Step 2: Receive multipliers (μ_j, λ_j) from neighbors and update

$$y_{\text{eq},i}^{k+1} = y_{\text{eq},i}^k + \sum_{j \in \mathcal{N}_i} (\lambda_i - \lambda_j), \quad y_{\text{in},i}^{k+1} = y_{\text{in},i}^k + \sum_{j \in \mathcal{N}_i} (\mu_i - \mu_j)$$

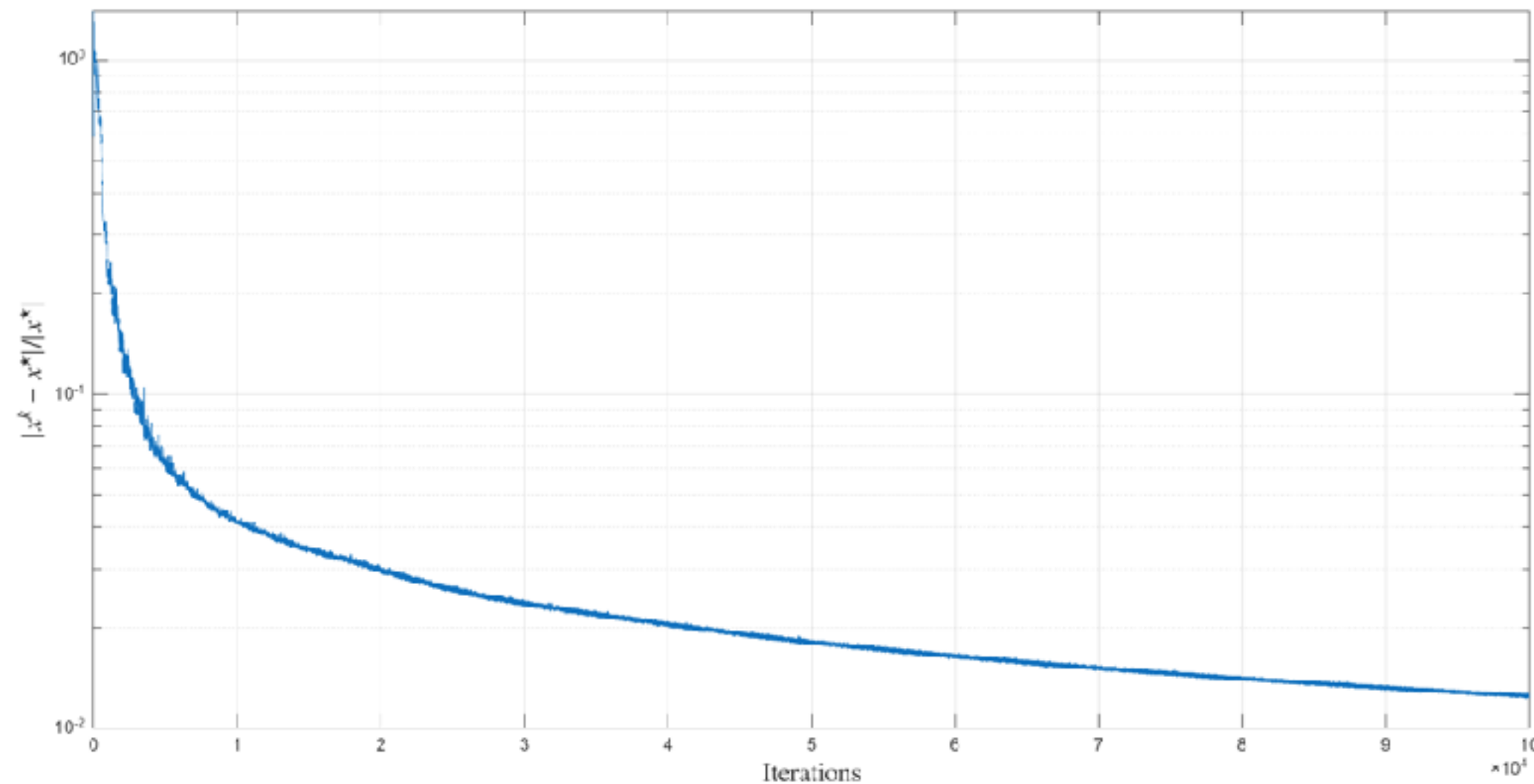
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Numerical Simulation

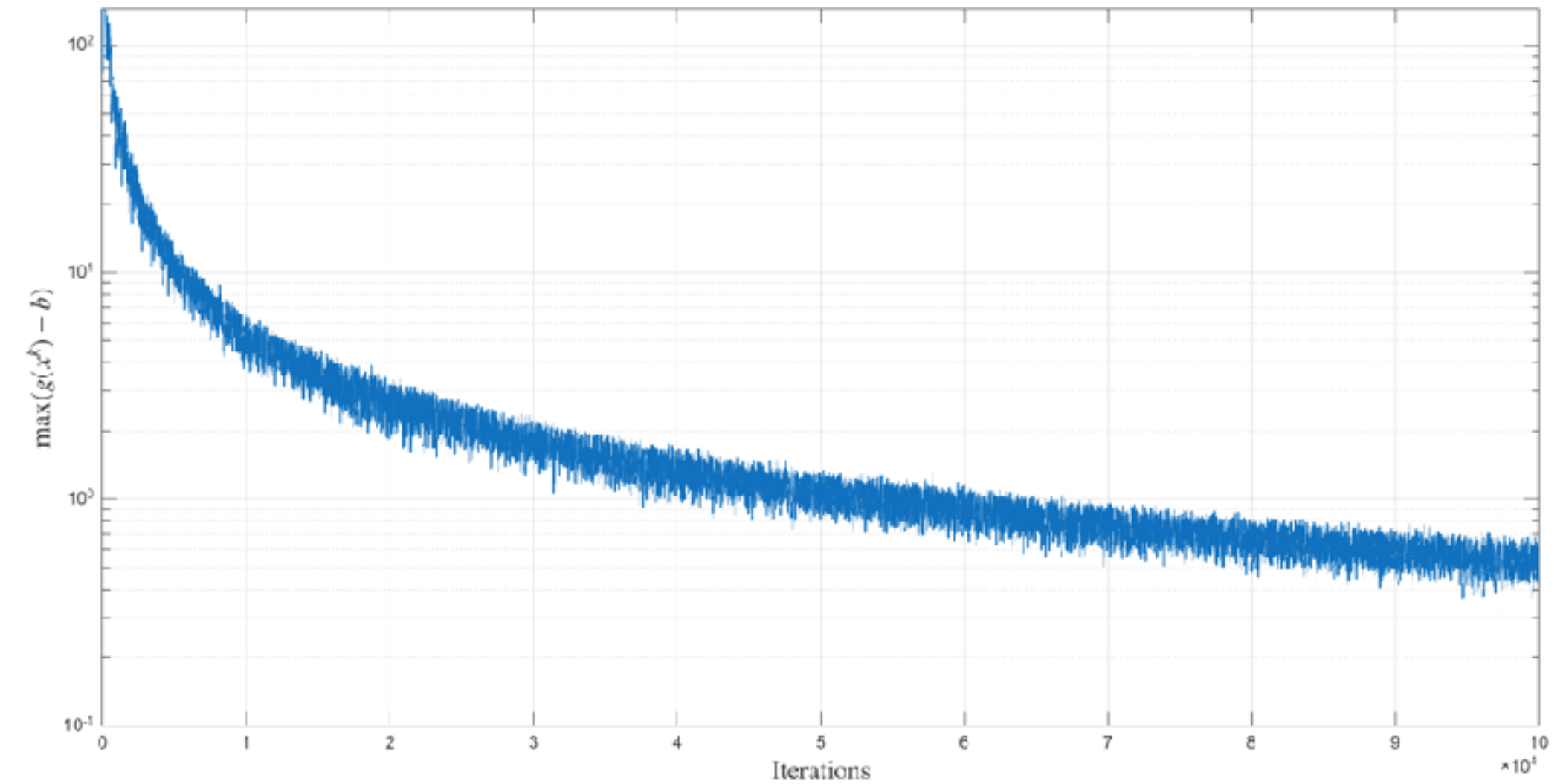
Setup: for $N = 8$ agents and $T = 96$

- $L = 3$ renewable sources
- $H = 2$ turbines element
- $K = 1$ storage element

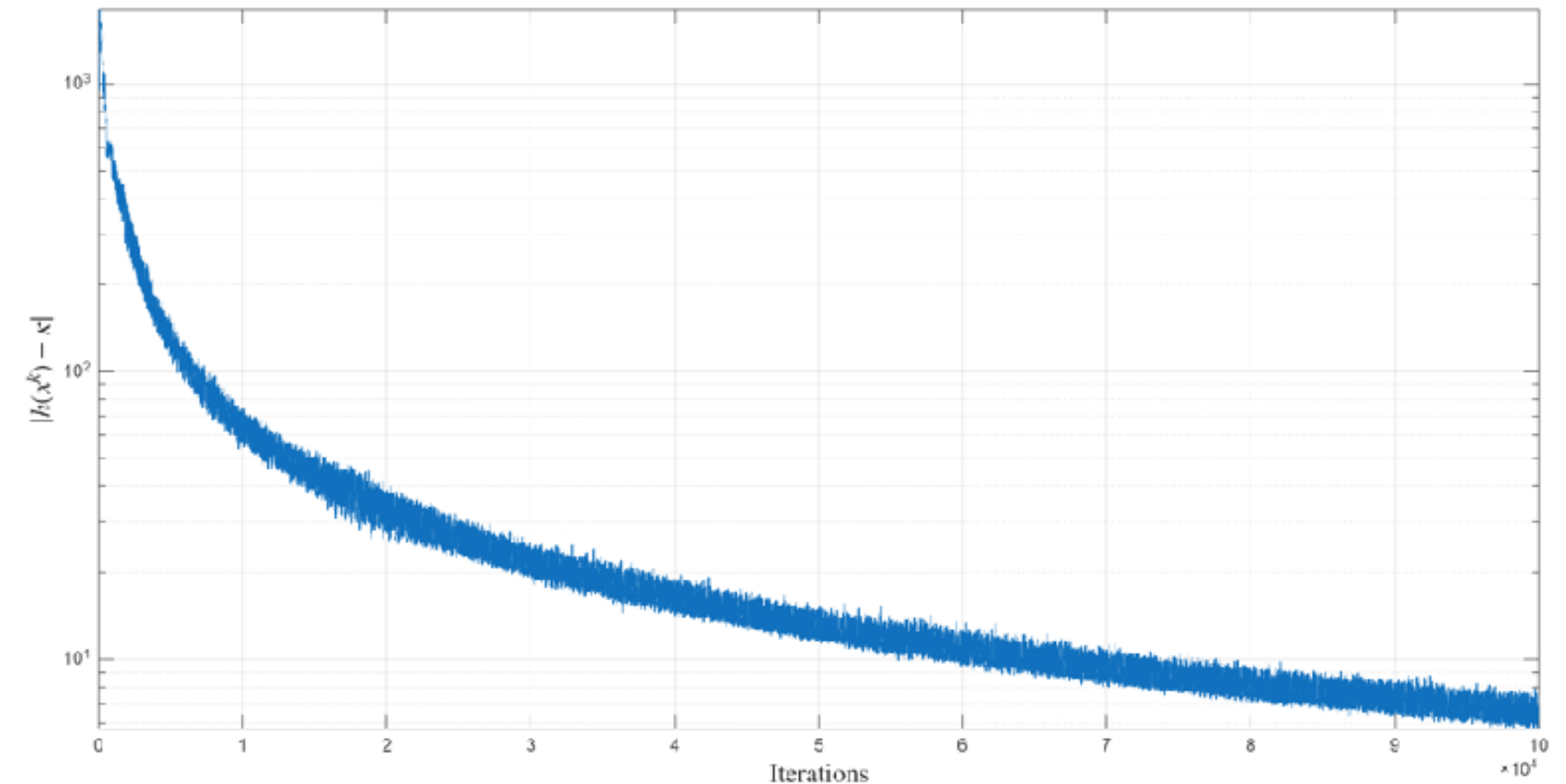
Relative distance from the solution x^*



Thermal energy violation

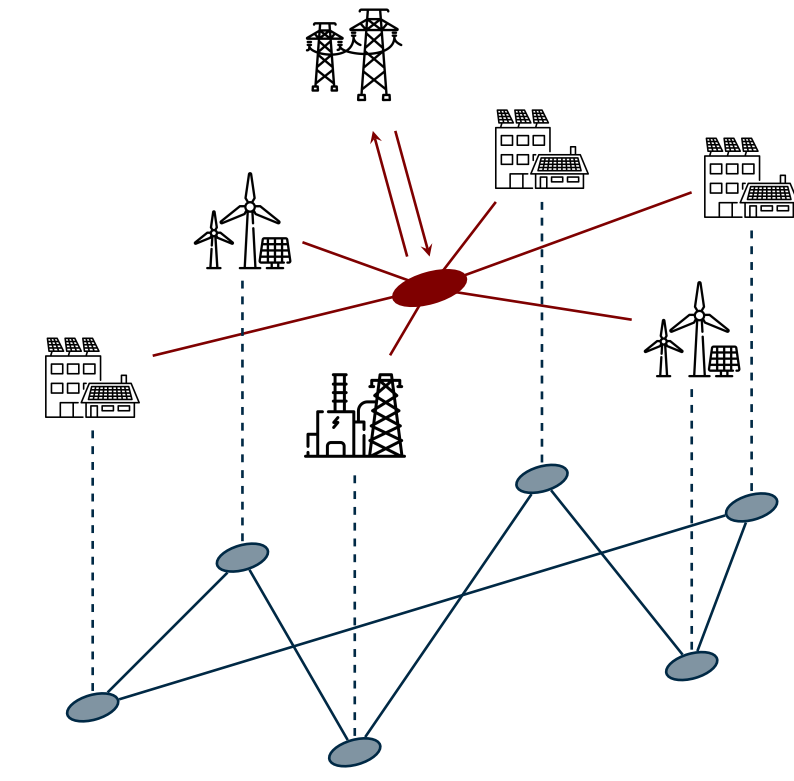


Power balance violation

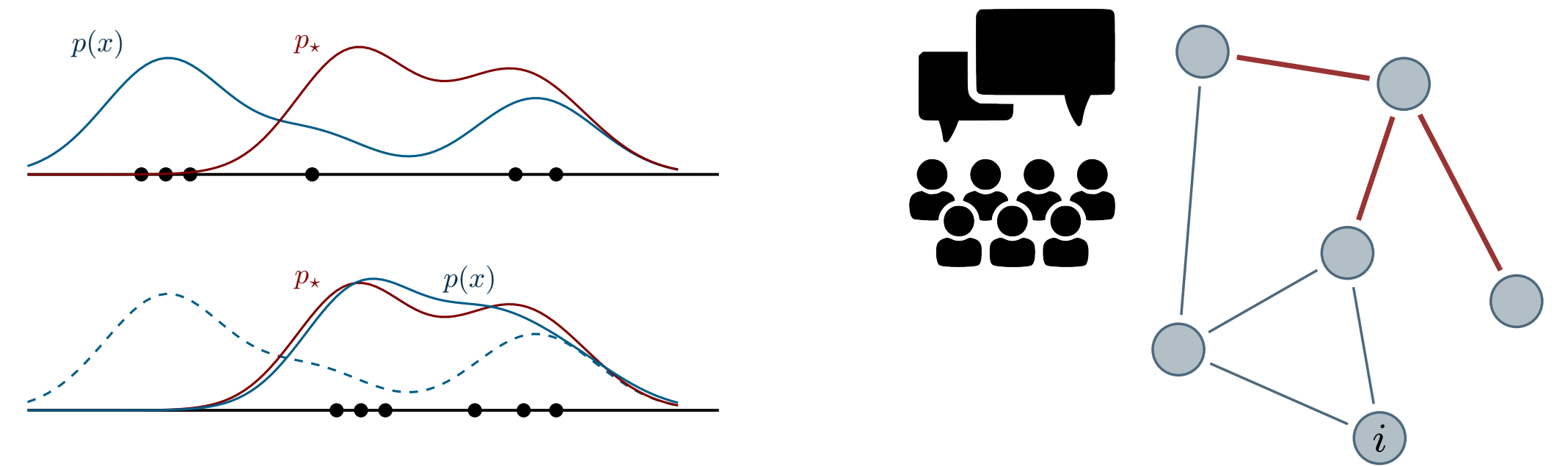


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Distributed Algorithm for Coordination in Energy Communities



Distributed learning and optimization in aggregative settings



Application of distributed optimization to microgrid testbed

