Balancing Service Provision via Aggregation of Flexible Prosumers

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November 28, 2025

ECODREAM Final event – University Federico II, Naples

Outline

- Context and motivation
- Problem formulation
- Existing approaches
- Proposed method
- Simulation results
- Conclusions

Context – net zero target

Global warming is causing climate changes and jeopardizing the habitability of the planet.

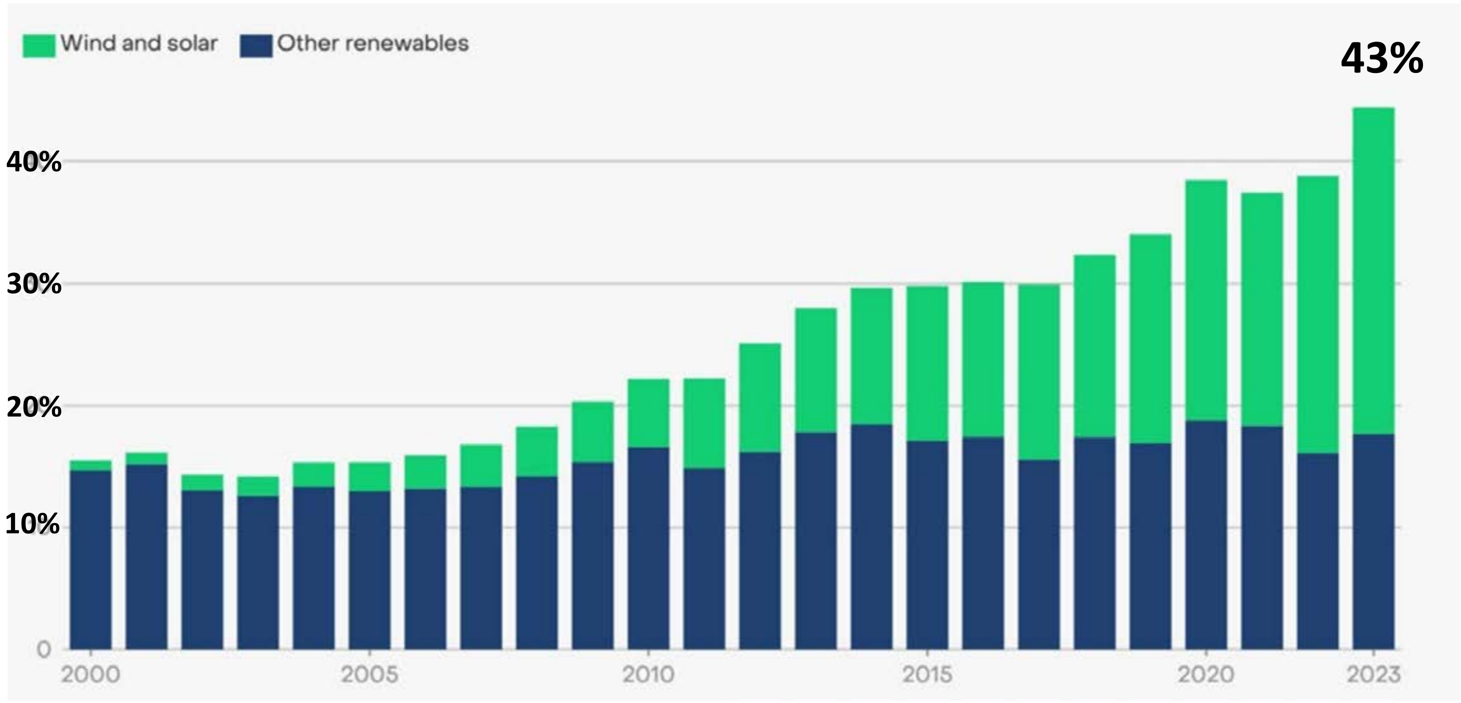
Paris Agreement in 2015: temperature growth below 1.5°C with respect to pre-industrial levels to mitigate the most severe effects of climate change.

EU Target (European Green Deal, 2019):
1st carbon neutral continent by 2050

Intermediate “Fit to 55” goal: reduction of the greenhouse gas emissions of 55% by 2030, with respect to the levels in 1990.

Context – energy transition

Renewable share [%] of EU electricity generation



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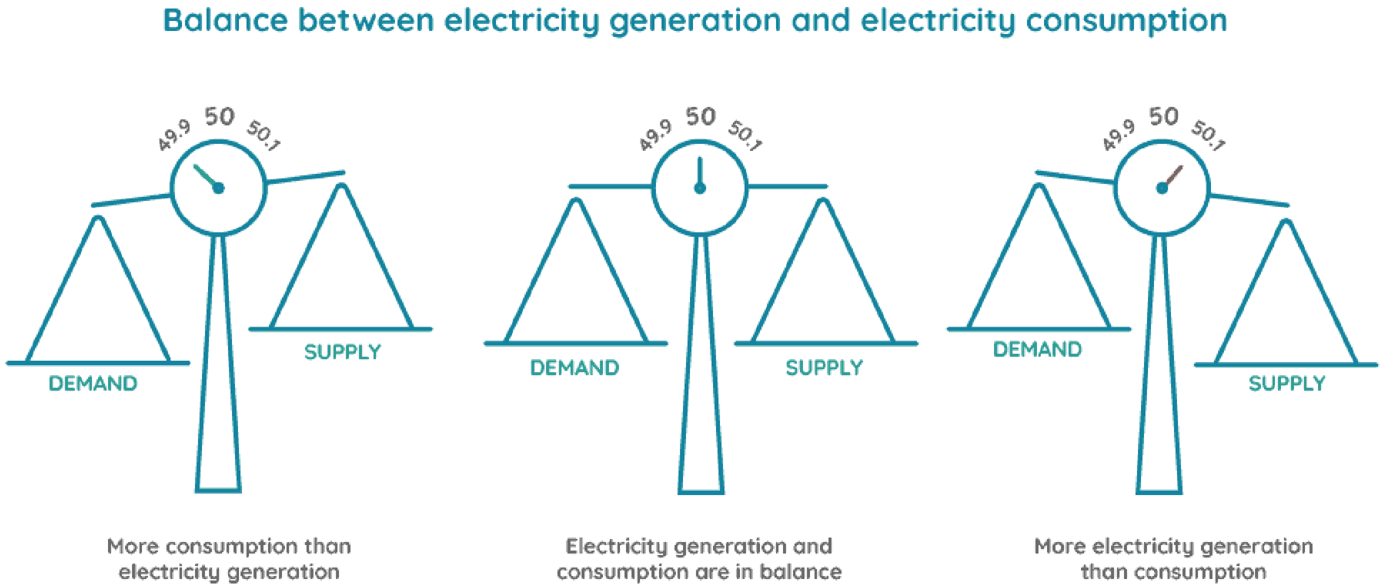
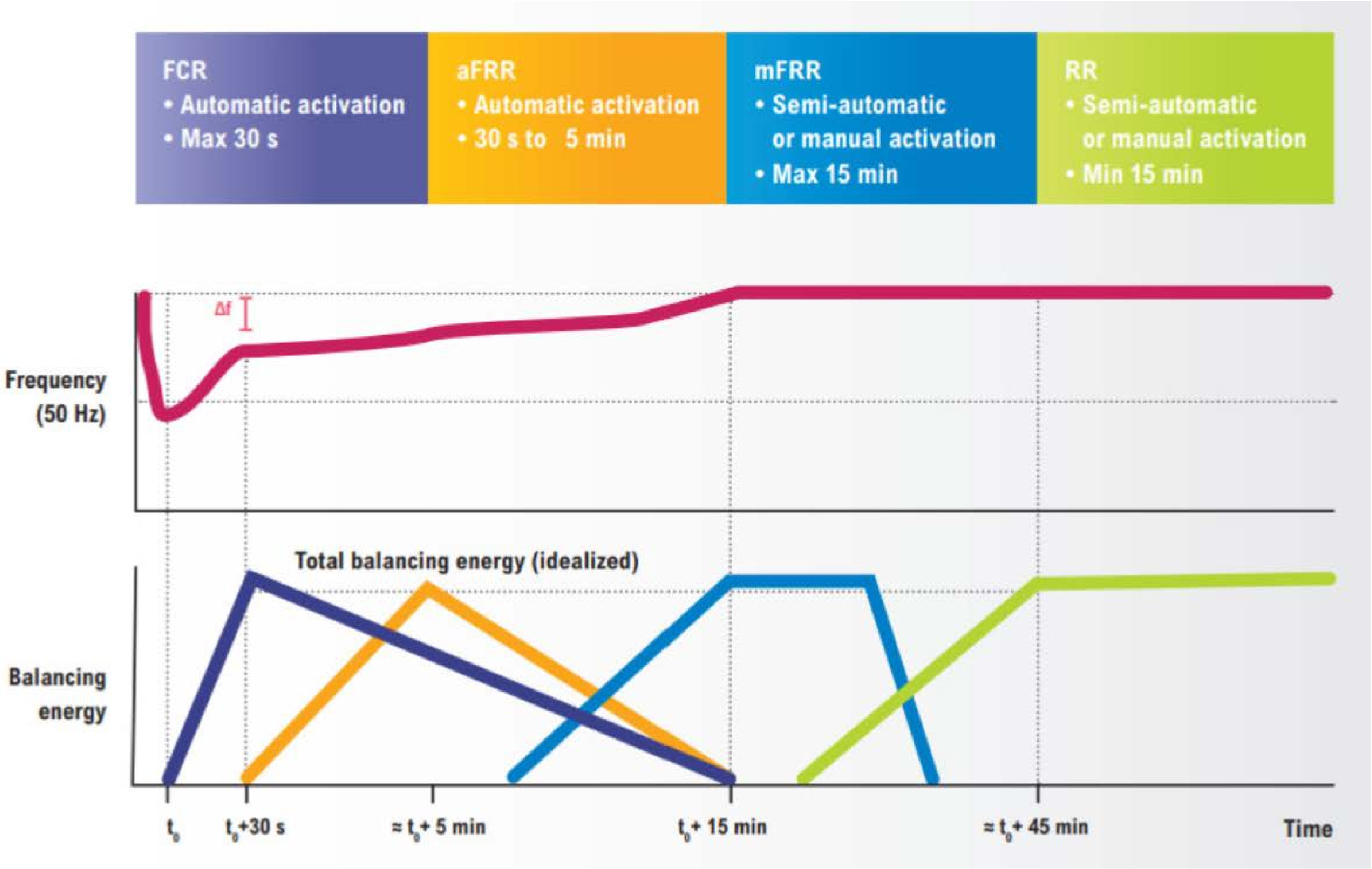
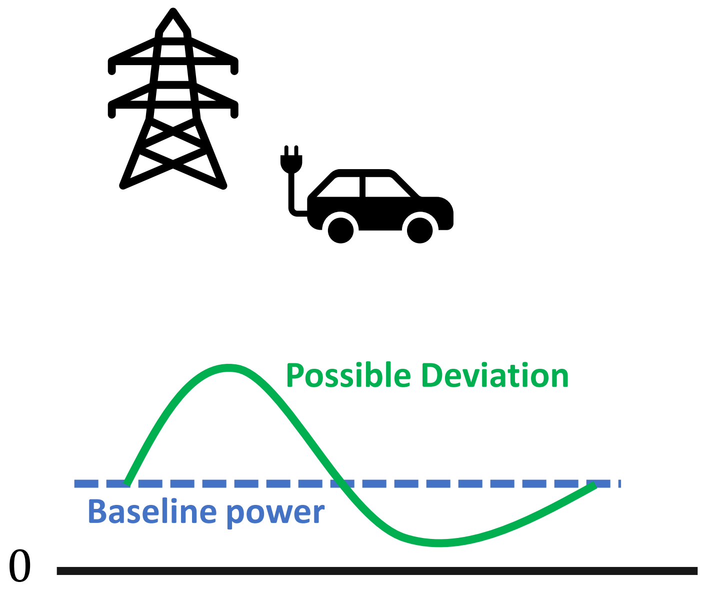


Context – balancing services



Balancing service via prosumers aggregation

A possible strategy consists of engaging end-users with energy storage, flexible loads, and production capabilities (prosumers) to actively provide balancing services to the grid.



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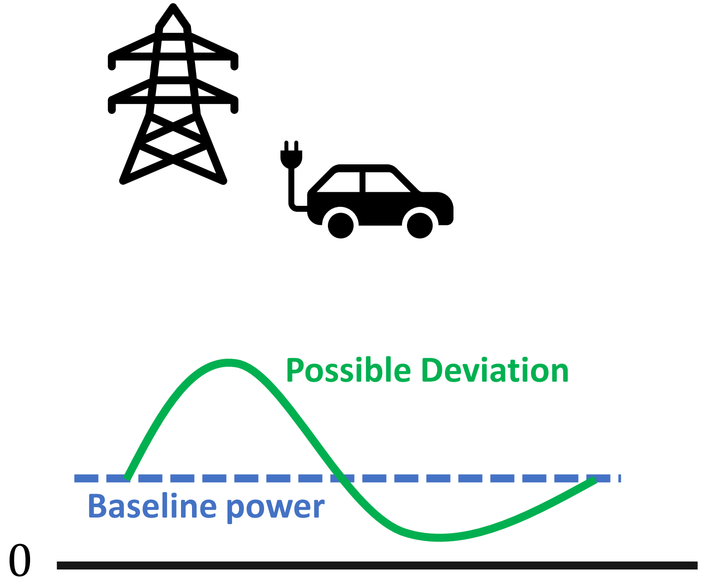
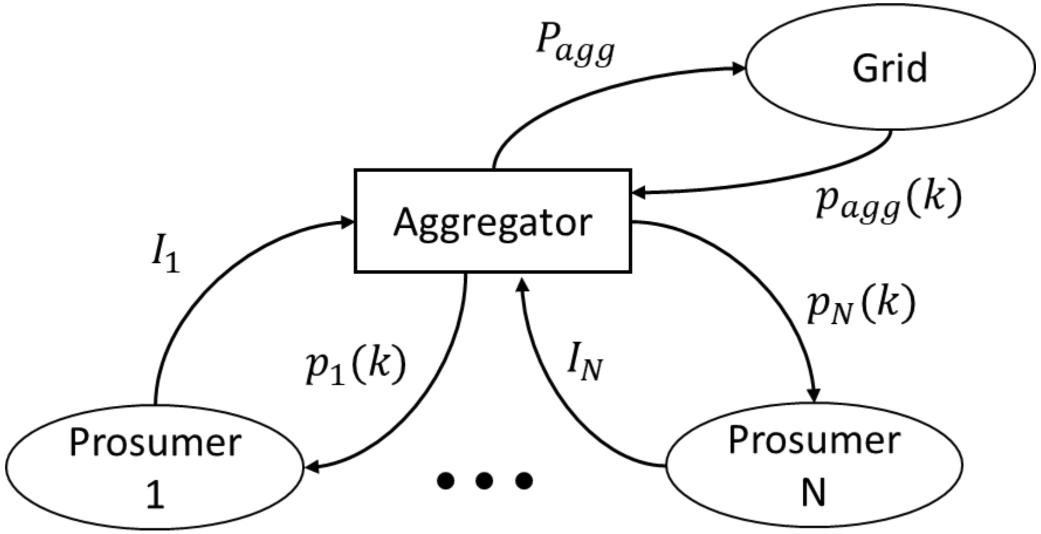
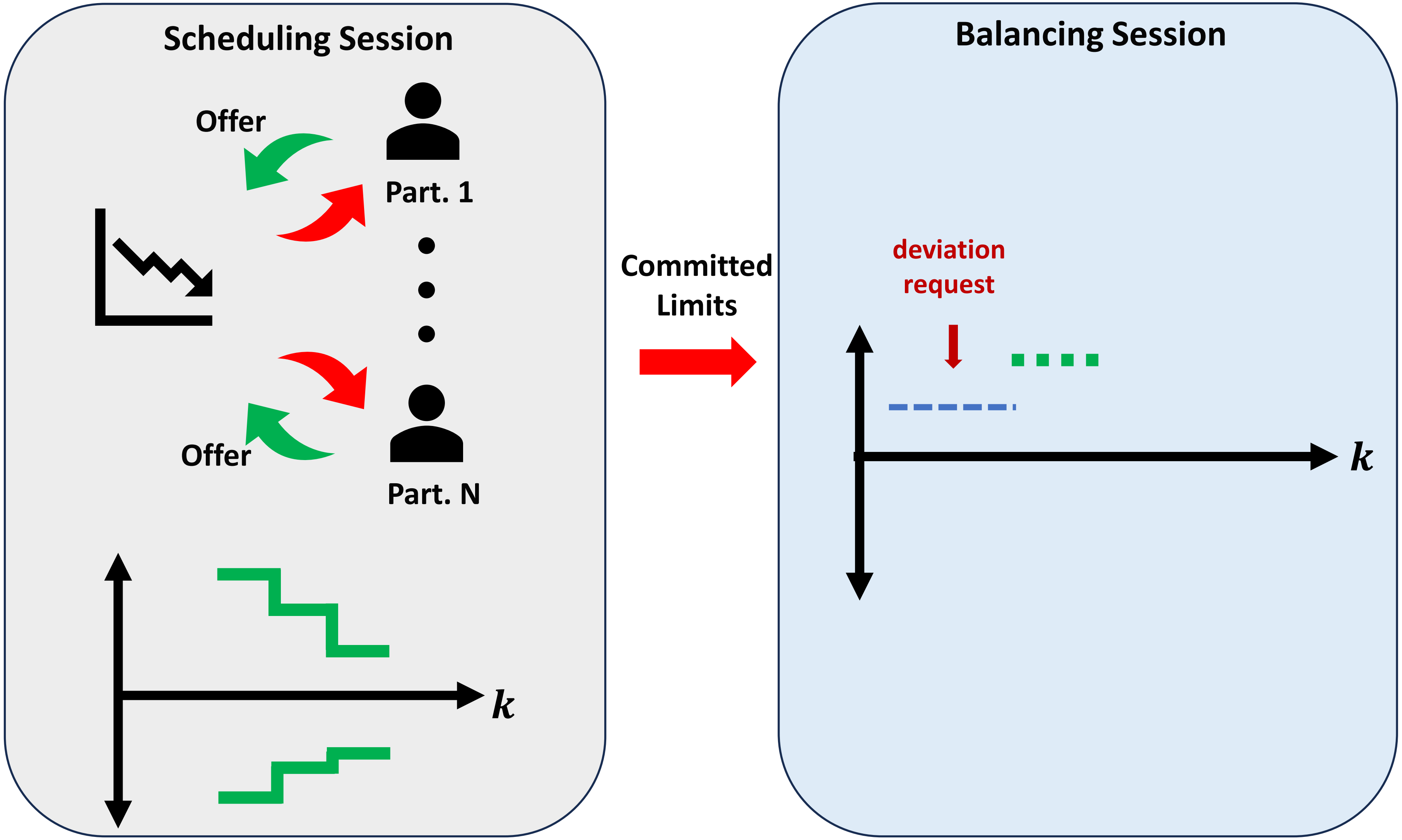




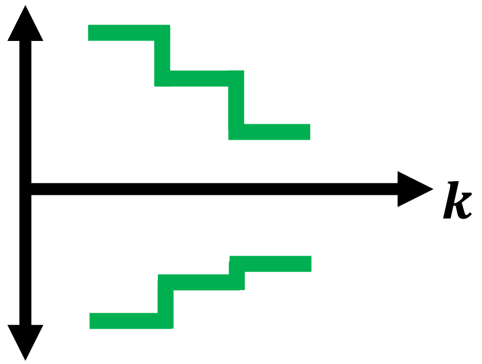
$p(k)$ is the average power during time slot k

Balancing service via prosumers aggregation



Balancing service via prosumers aggregation

Scheduling Session



Balancing Session

Main challenges for the aggregator:

Asses the upper and lower power flexibility limits per time slot, considering:

- prosumers availability
- local constraints
- possible network constraints.

Balancing service via prosumers aggregation

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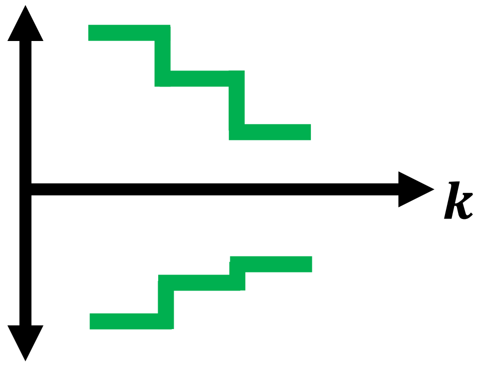

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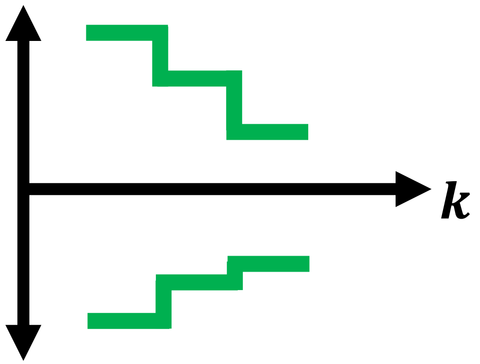
Respond to any admissible request by coordinating the prosumers, considering

- short time for dispatch.

Balancing service via prosumers aggregation

Scheduling Session



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- possible network constraints.

Respond to any admissible request by coordinating the prosumers, considering

- short time for dispatch.

In both sessions, privacy constraints might be present

Prosumers modeling

$\mathcal{T} = \{1, \ldots, M\}$ slot duration τ typically, 15 min

$p(k)$ is the average power during k

- $p(k) < 0$ **Provide**
- $p(k) > 0$ **Absorb**

Power Bounds

$$\ell_p(k) \leq p(k) \leq u_p(k)$$

Power Rate Bounds

$$\ell_r(k) \leq p(k) - p(k-1) \leq u_r(k)$$

Energy Dynamics

$$e(k+1) = \zeta e(k) + b_p p(k)$$

Stored Energy Bounds

$$\ell_e(k) \leq e(k) \leq u_e(k)$$

Different types of devices can be modelled

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Power Generator (PG)

$$-P_{max} \leq p(k) \leq 0, \quad P_{max} > 0$$

POLITECNICO MILANO 1863

Prosumers modeling

DEIB

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Electric Battery (EB)

$$b_p = \tau$$

$$e(k+1) = \zeta e(k) + \tau p(k)$$

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Stored Energy Bounds

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Electric Vehicles (EV)

$$b_p = \tau$$

$$e(k+1) = \zeta e(k) + \tau p(k)$$

By fixing $\ell_e(M) = e_f$

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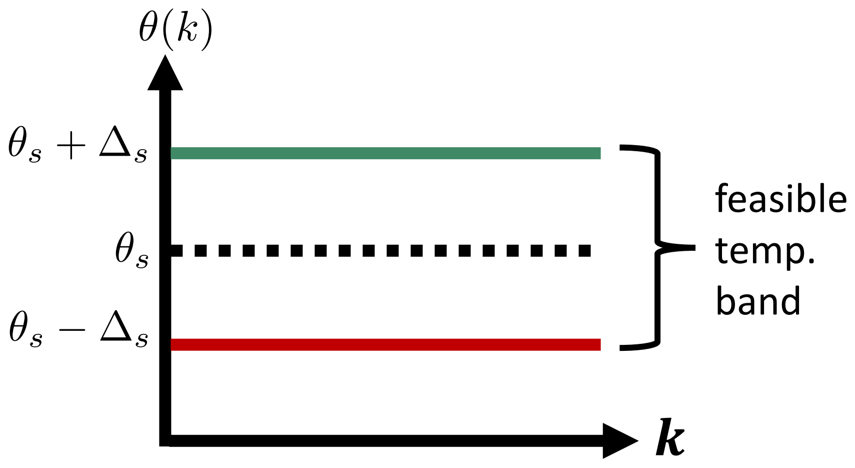
Thermostatically Controlled Load (TCL)



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Since all constraints are linear and the energy dynamics can be unrolled in terms of the power, they form a convex polytopic set in the power space.

$$\mathcal{P} = \{p \in \mathbb{R}^{|\mathcal{T}|} : \ Fp \le h\}$$

Prosumers flexibility

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$$\ell_e \leq e(k) \leq u_e$$
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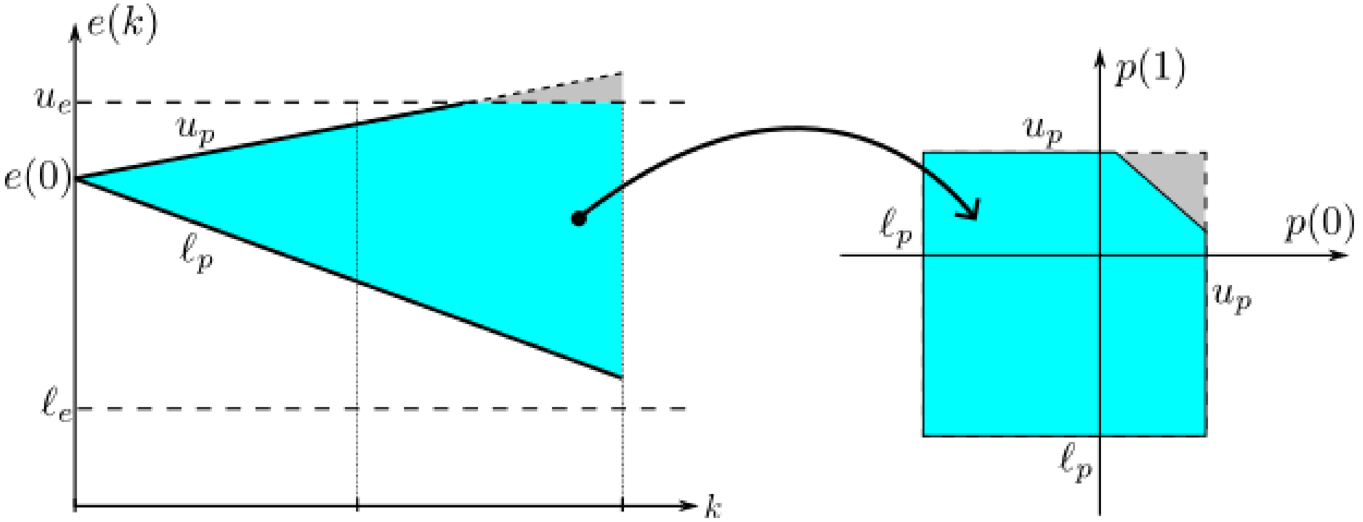

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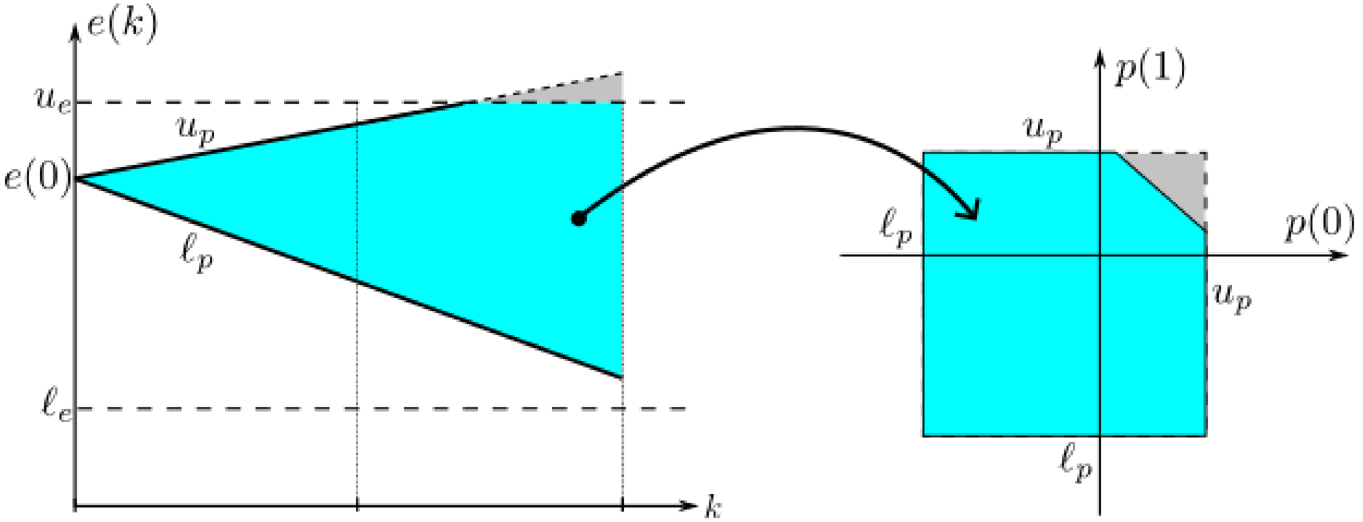

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baseline

$p = \bar{p} + \delta$ → **deviation**

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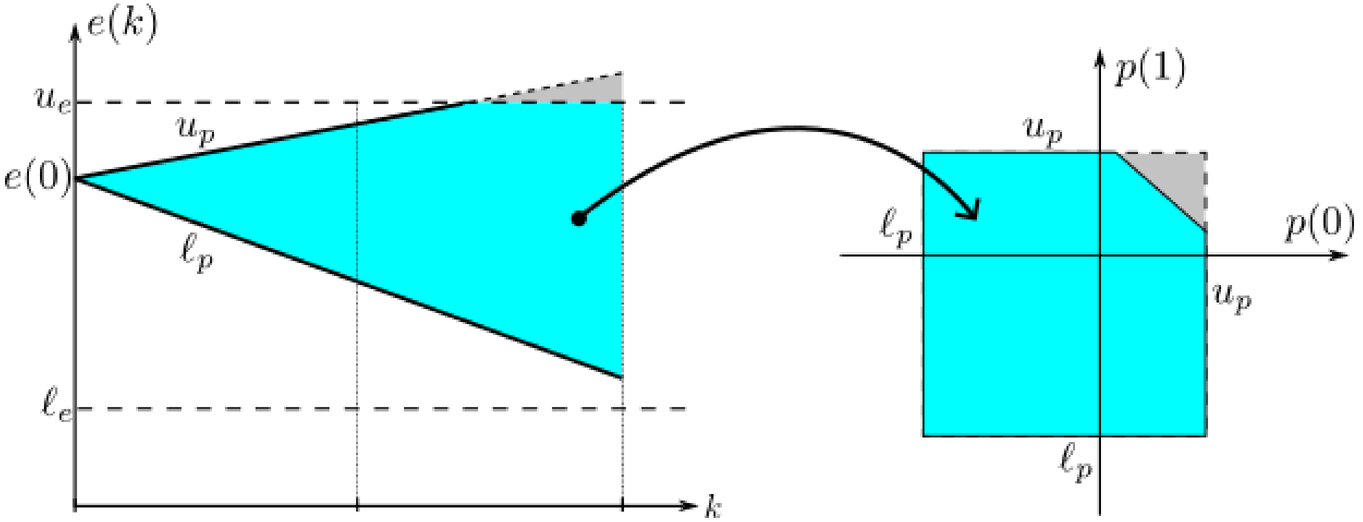

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Prosumer's Flexibility Set

$$\Delta := \{\delta \in \mathbb{R}^{|\mathcal{T}|} : \; p = \bar{p} + \delta \in \mathcal{P}\}$$
$$:= \{\delta \in \mathbb{R}^{|\mathcal{T}|} : \; F\delta \leq h - F\bar{p}\}$$

Aggregate flexibility set

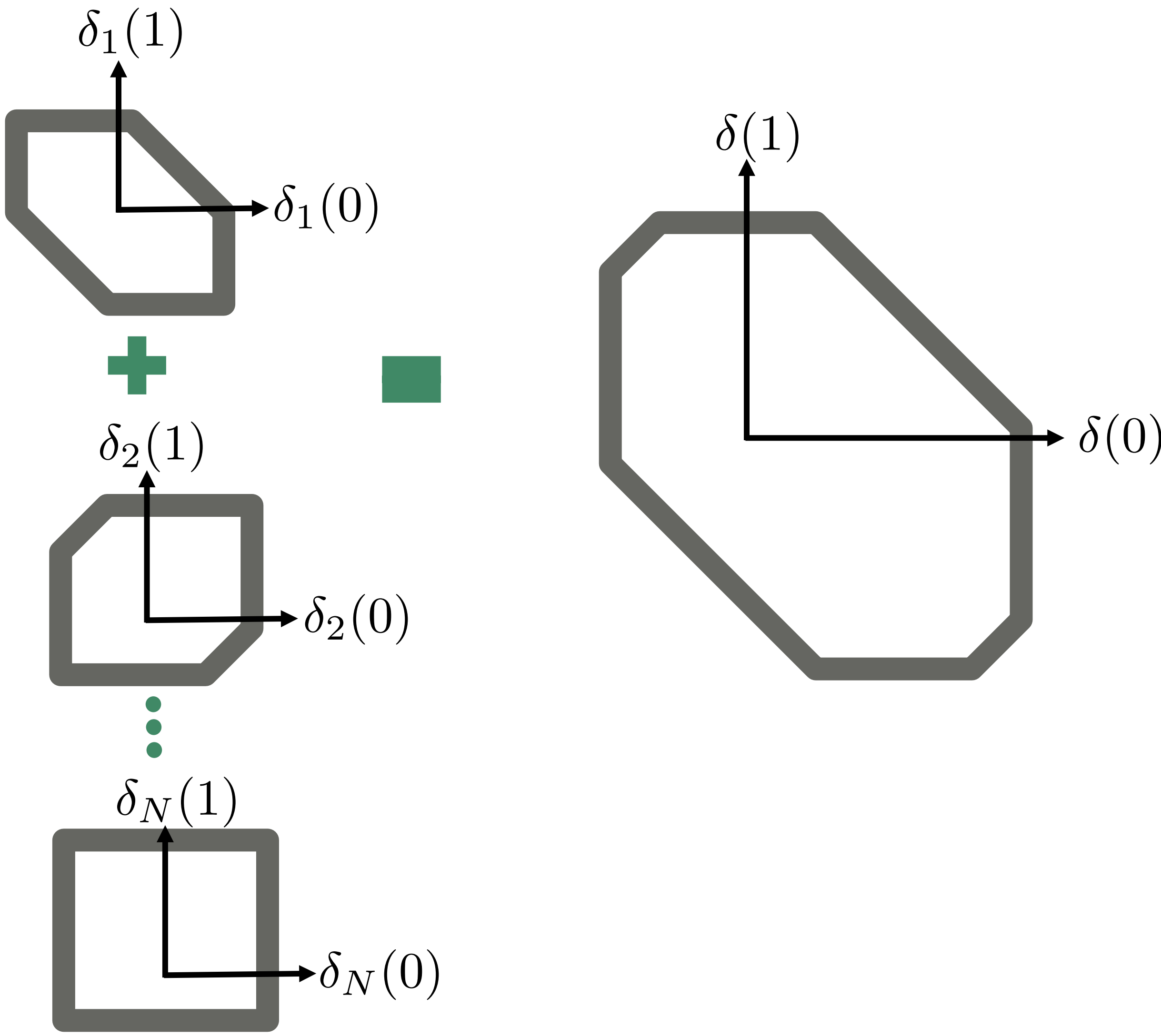
Consider a pool of prosumers indexed by *i* with flexibility sets:

$$\Delta_i,\ i \in \mathcal{I}$$

$$\mathcal{I} = \{1, 2, \ldots, N\}$$

The aggregate flexibility set is given by:

$$\Delta = \left\{\delta \in \mathbb{R}^M :\ \delta = \sum_{i\in\mathcal{I}} \delta_i,\ \delta_i \in \Delta_i,\ i \in \mathcal{I}\right\}$$



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which is the Minkowski sum

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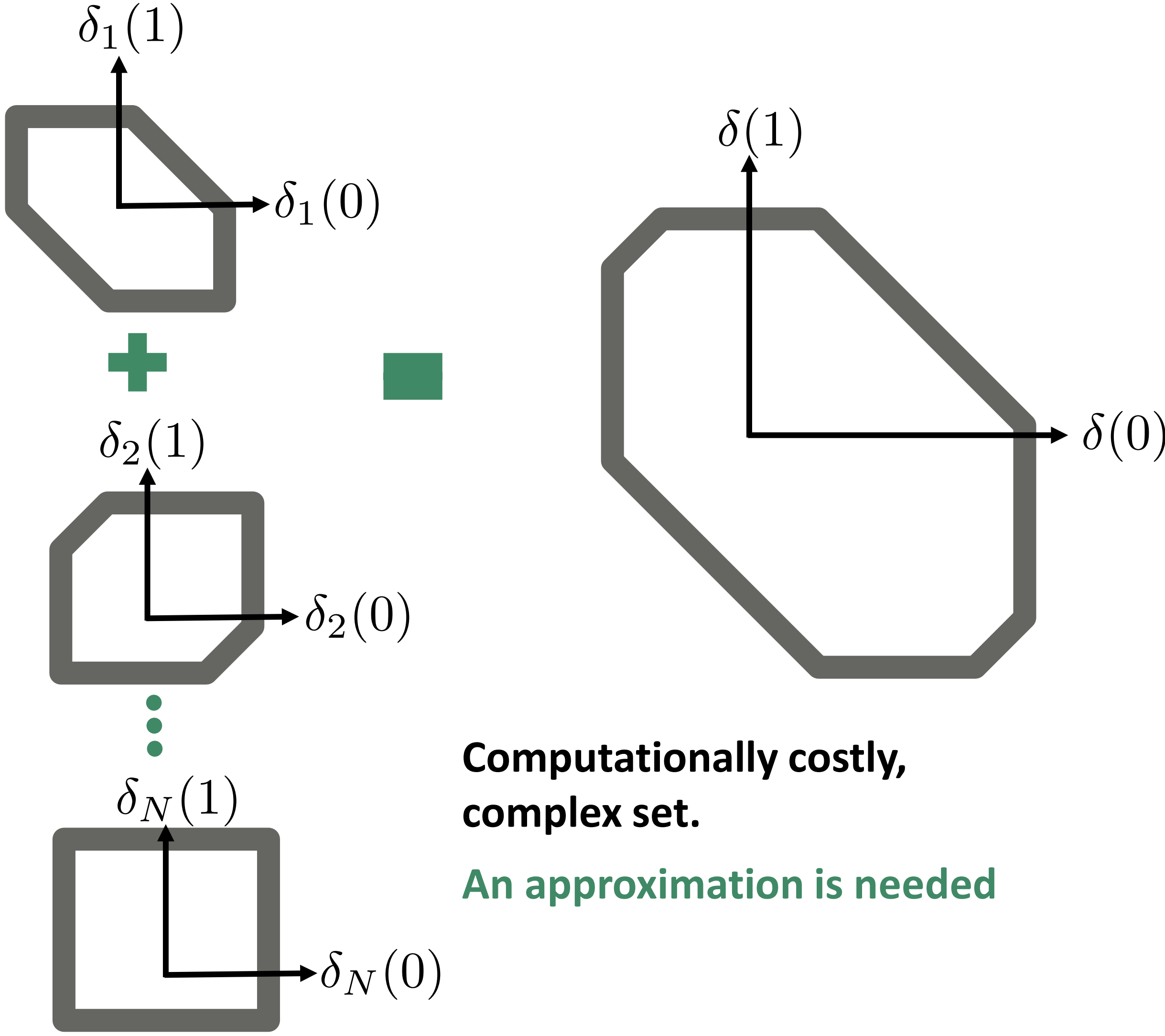

Computationally costly, complex set.

An approximation is needed

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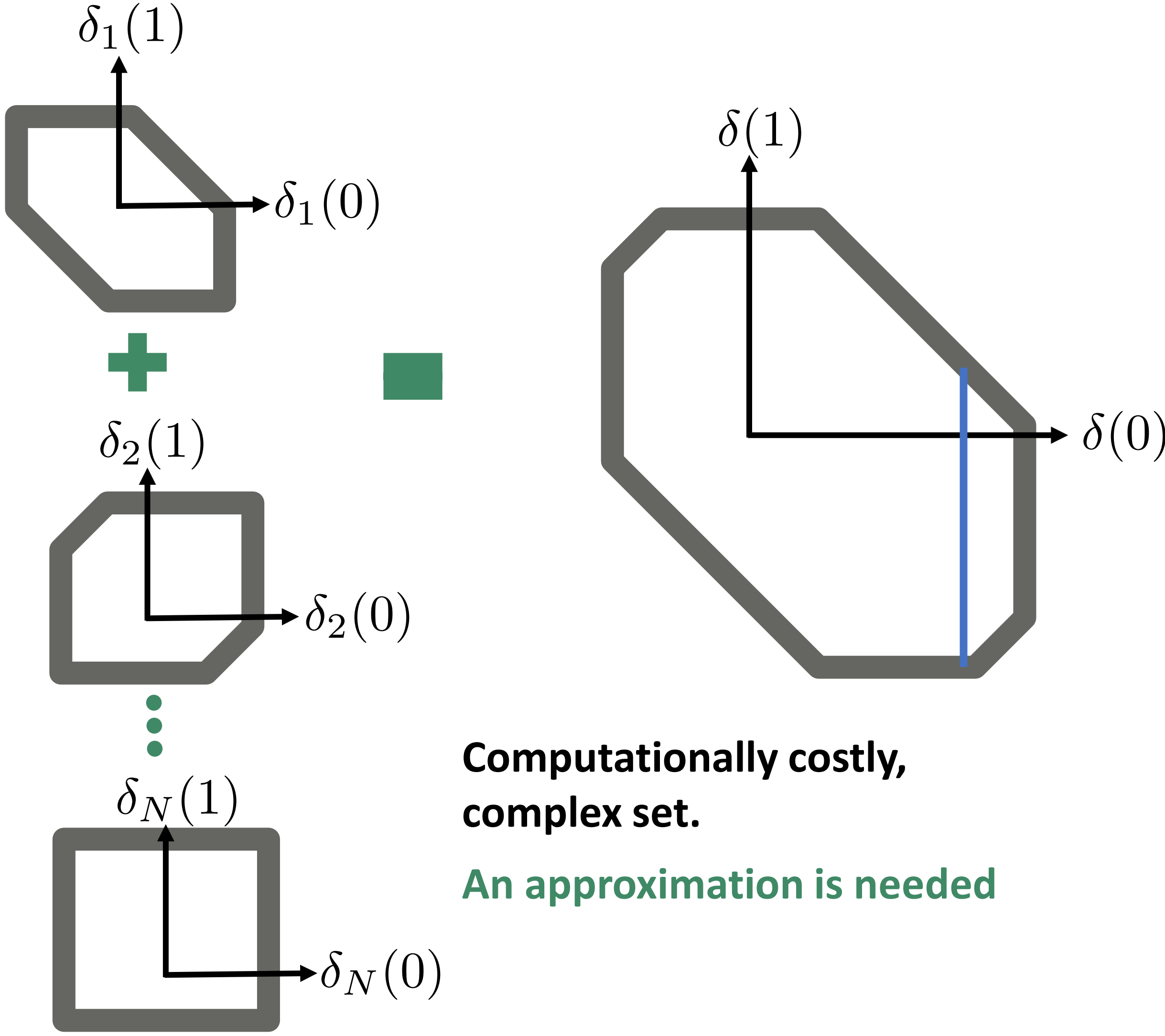

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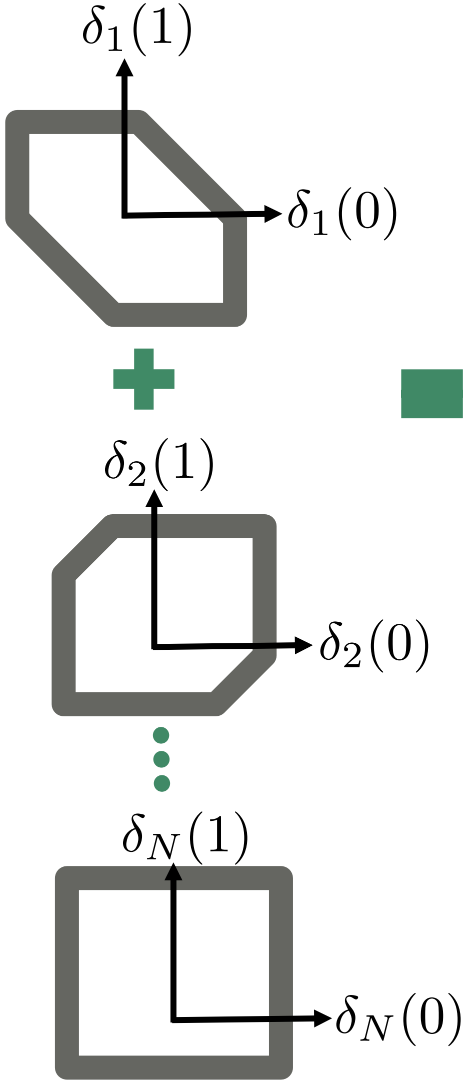

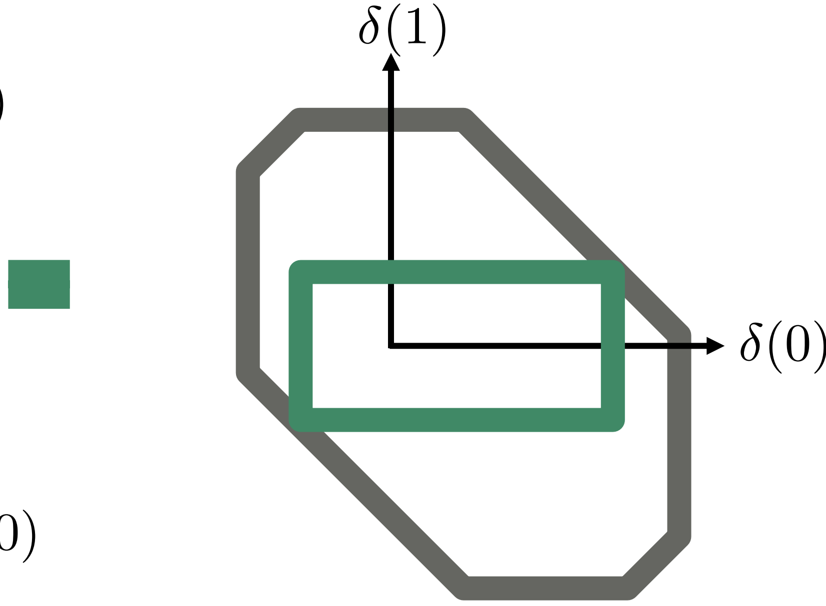

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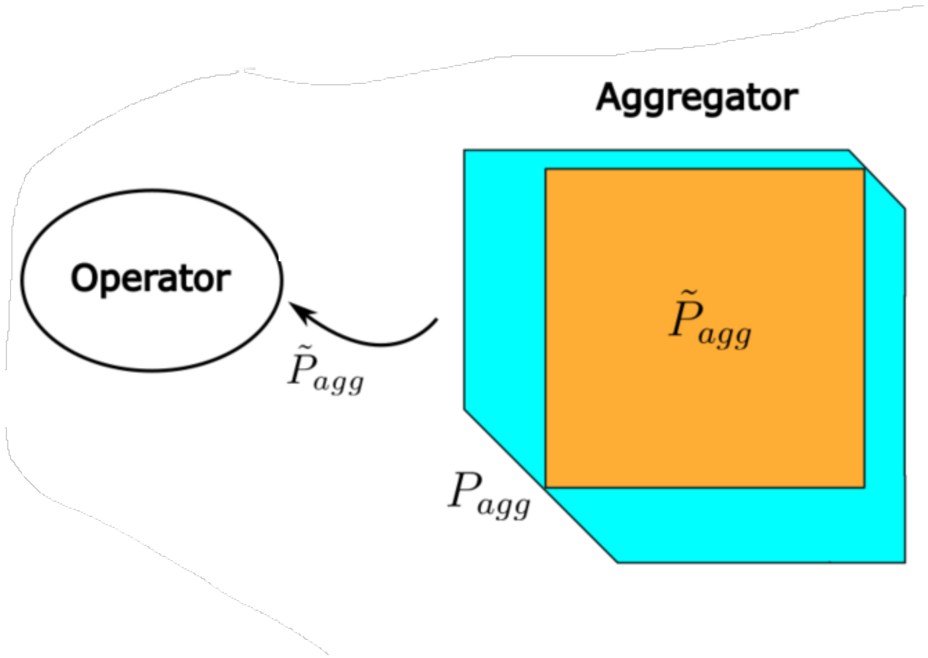
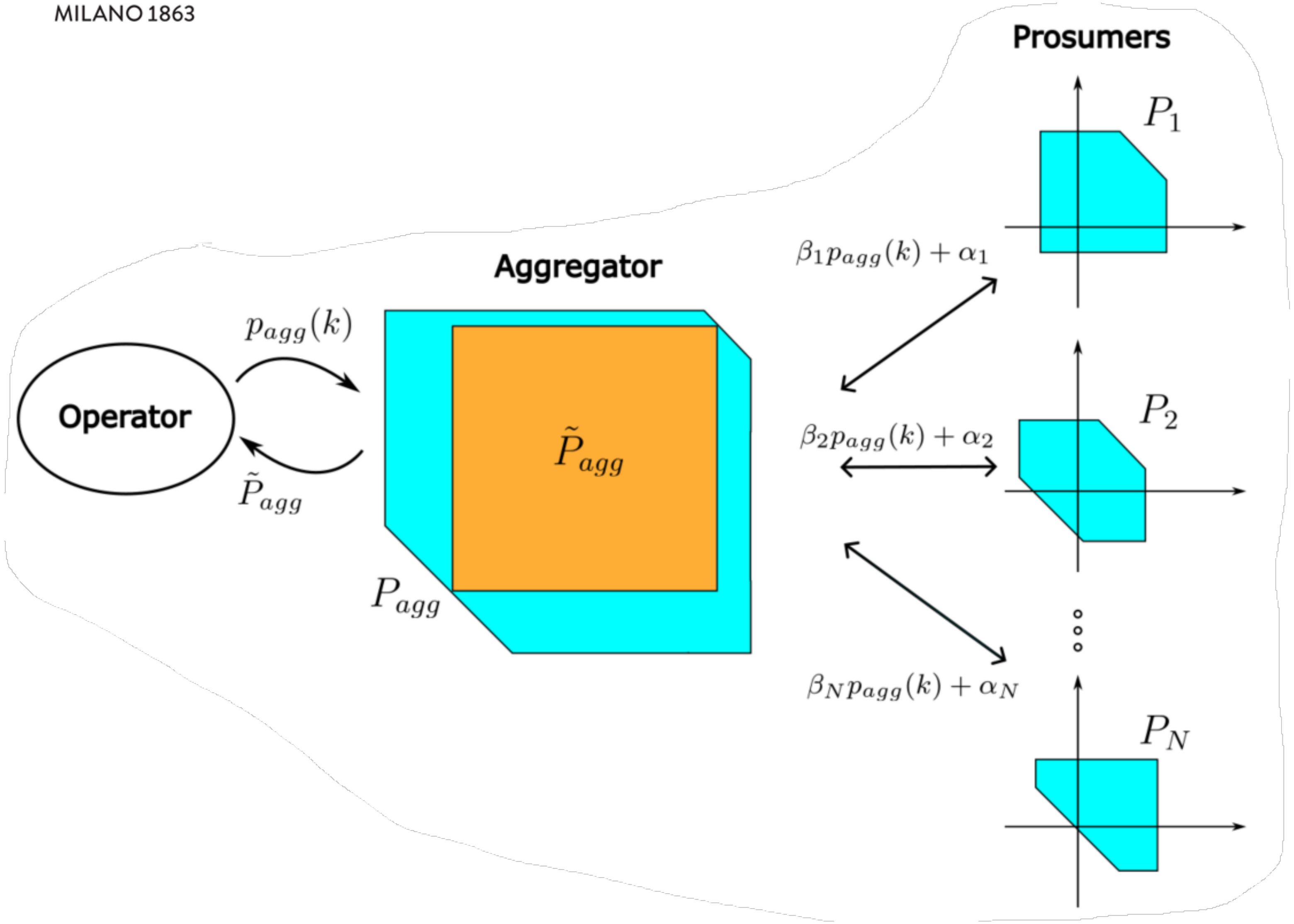
Aggregate



Aggregate & disaggregate



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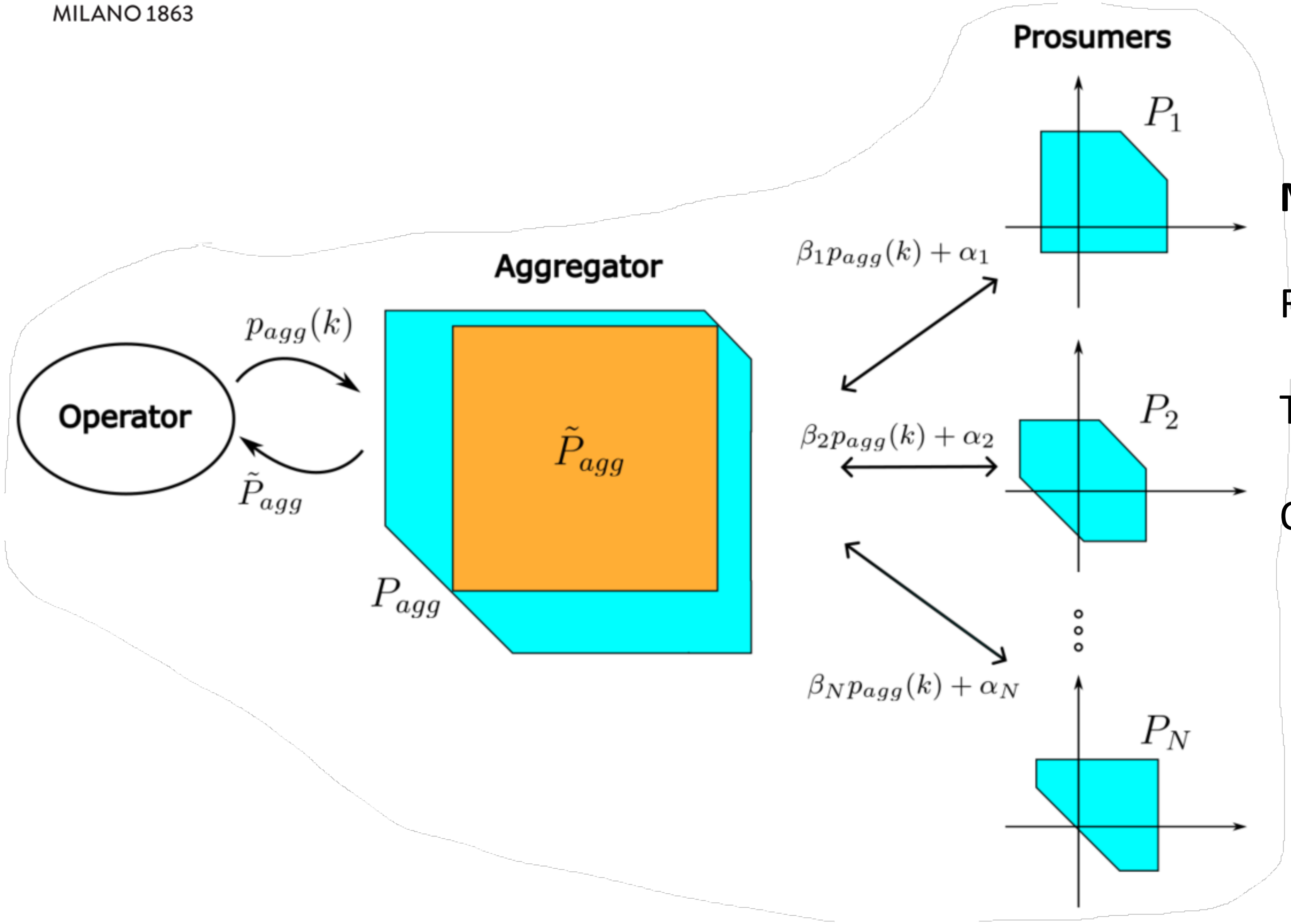

Methods can be divided into:

Flexibility assessment methods **FA**

Two-stage balancing provision methods **2-BP**

One-stage balancing provision methods **1-BP**

Existing methods

Method	Category	Box-shaped flexibility set	Possible joint baseline optimization	Parallel implementation	Network constraints	Not always available loads	Privacy preservation
Enhanced Sufficient Battery Model	FA				✓		
Ellipsoidal-shaped Projection via Linear Rule	FA		✓			✓	
Zonotope-based Approximation	2-BP	✓	✓	✓		✓	✓
Ellipsoidal-shaped Projection via ARO	2-BP		✓		✓	✓	
Box-shaped Projection via ARO	2-BP	✓	✓		✓	✓	
Battery Homothets Approximation	1-BP	✓	✓	✓			✓
Cuboid Decomposition with Stage 0	1-BP	✓	✓	✓			✓
Cuboid Decomposition with Multiple Stages	1-BP		✓	✓			✓
Homothetic Polytope-shaped Projection via Linear Rule	1-BP		✓	✓		✓	
Generalized Battery Model	1-BP						

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Battery homothets approximation

A homothet of   is defined as the following affine transformation:

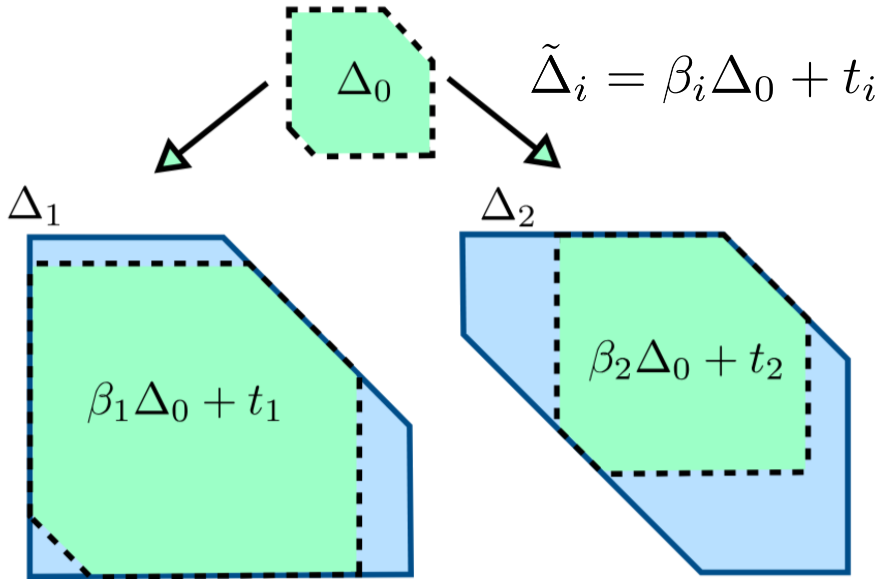
$$\Delta_i = \beta\Delta_0 + t = \{\delta_i \in \mathbb{R}^M : \delta_i = \beta\delta + t,\ \delta \in \Delta_0\}$$

$$\beta \in \mathbb{R}_+ \quad t \in \mathbb{R}^M$$

If two sets are homothets of the same polytopic set Δ_0 their Minkowski sum is given by:

$$\Delta_i \oplus \Delta_j = (\beta_i + \beta_j)\Delta_0 + (t_i + t_j)$$

Prototype Set (TCL)



Zhao, L., Zhang, W., Hao, H., Kalsi, K. (2017). A Geometric Approach to Aggregate Flexibility Modeling of Thermostatically Controlled Loads. *IEEE Transactions on Power Systems*, *32*(6), 4721–4731.

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Main contribution

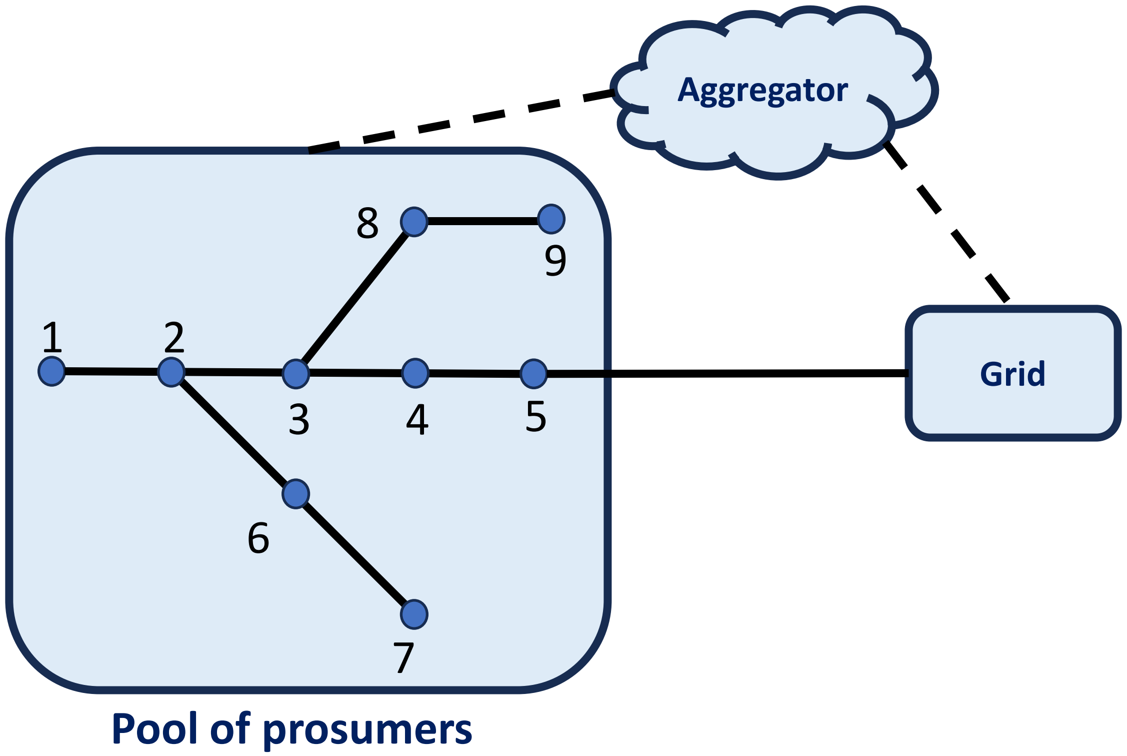
We propose a flexible and comprehensive **1-BP** method accounting for:

1. Box-shaped flexibility set & disaggregation policy.
2. Joint optimization of the baseline profile.
3. Inclusion of distribution network constraints
4. Possibility of parallelizing the computation while addressing network constraints.
5. Enhancing the aggregation of multiple prosumers by:
    - preserving the privacy of local information.
    - including loads with limited time availability.

Problem formulation

$$i \in \mathcal{I} = \{1, 2, \ldots, N\}, \;\; \mathcal{T} = \{1, \ldots, M\}$$

$$\mathcal{P}_i = \{p_i \in \mathbb{R}^M \; : \;\; F_i p_i \leq h_i\}$$



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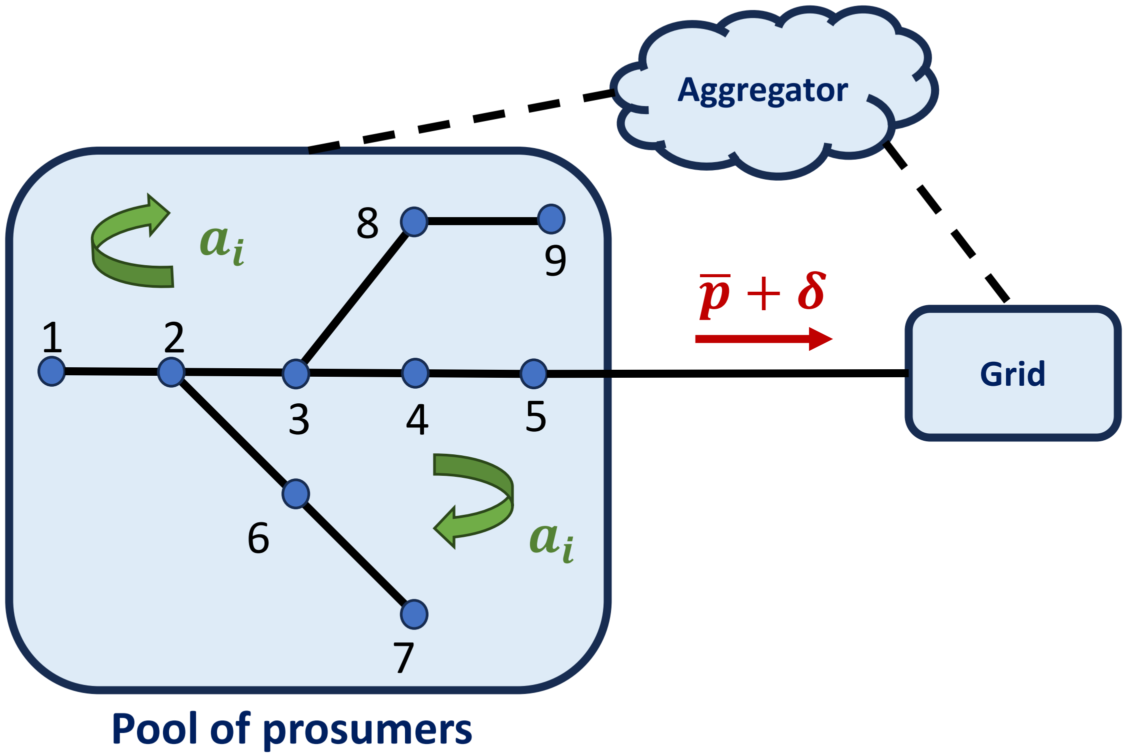
baseline

$p_i = \bar{p}_i + \delta_i$ → deviation

$\bar{p}_i = g_i + a_i$ → internal

external



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$$p_i = \bar{p}_i + \delta_i$$ → deviation

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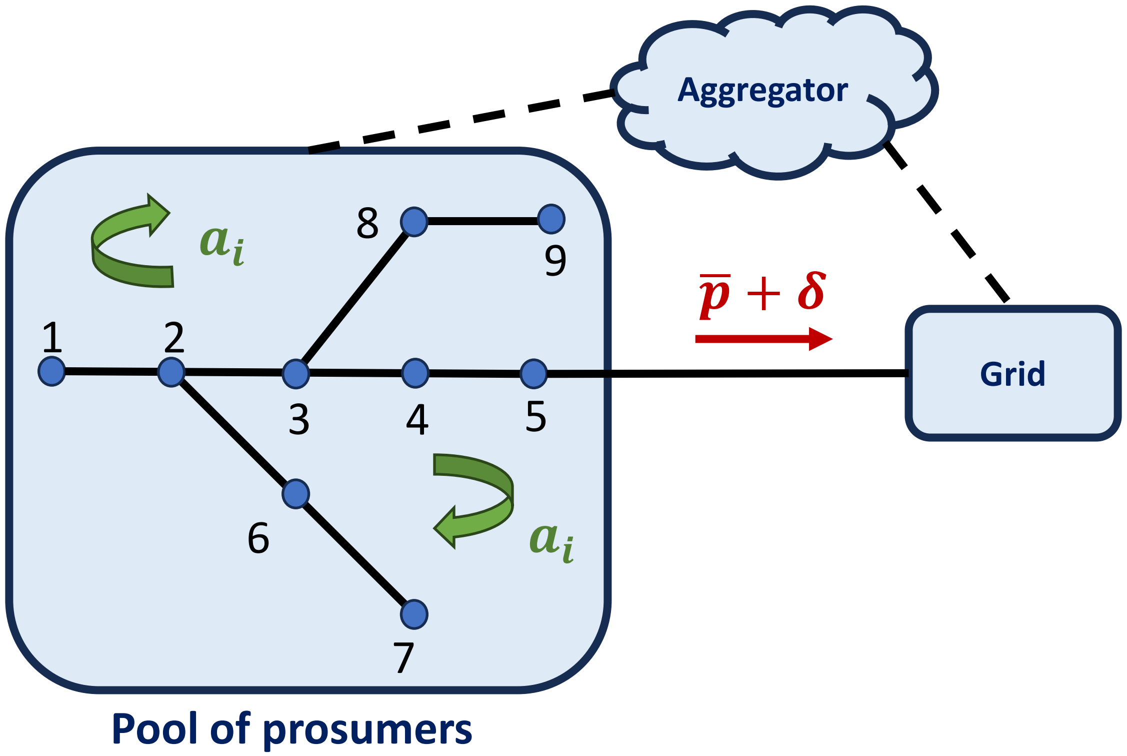
external

$$\sum_{i \in \mathcal{I}} \delta_i = \delta.$$

$$\sum_{i \in \mathcal{I}} a_i = 0$$

} Power Balance

$$\sum_{i \in \mathcal{I}} \bar{p}_i = \sum_{i \in \mathcal{I}} g_i = \bar{p}$$



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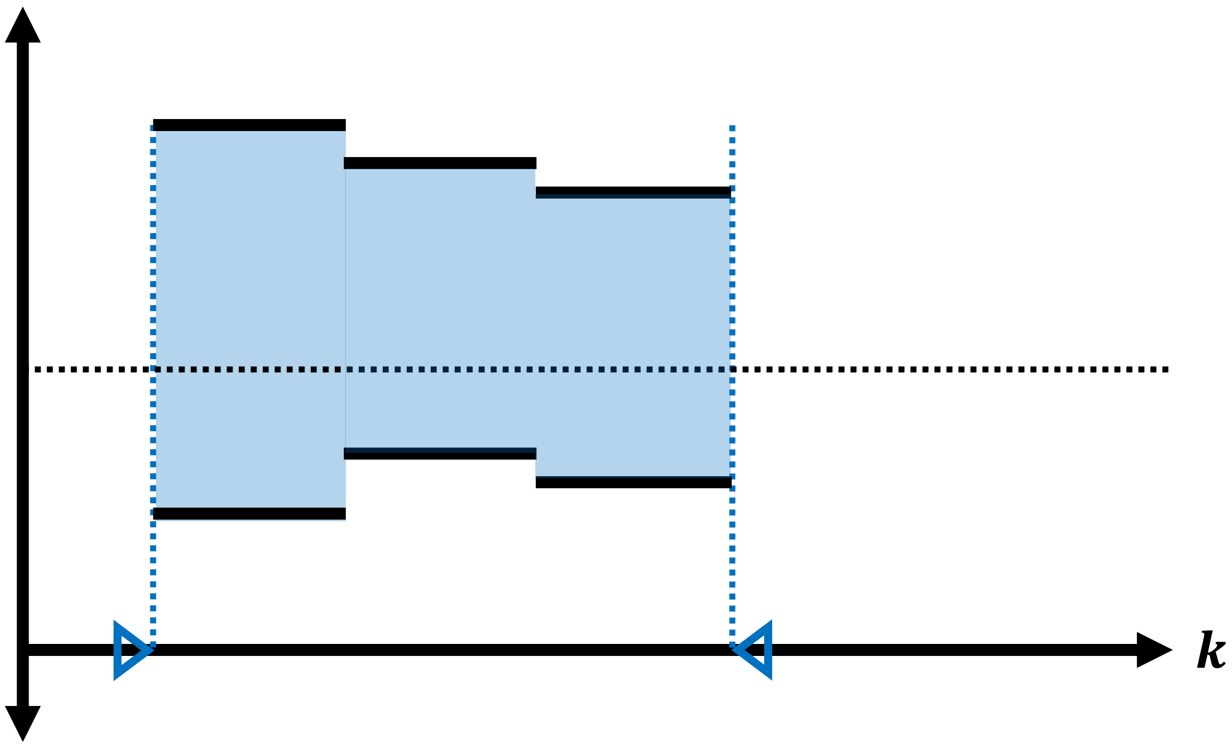
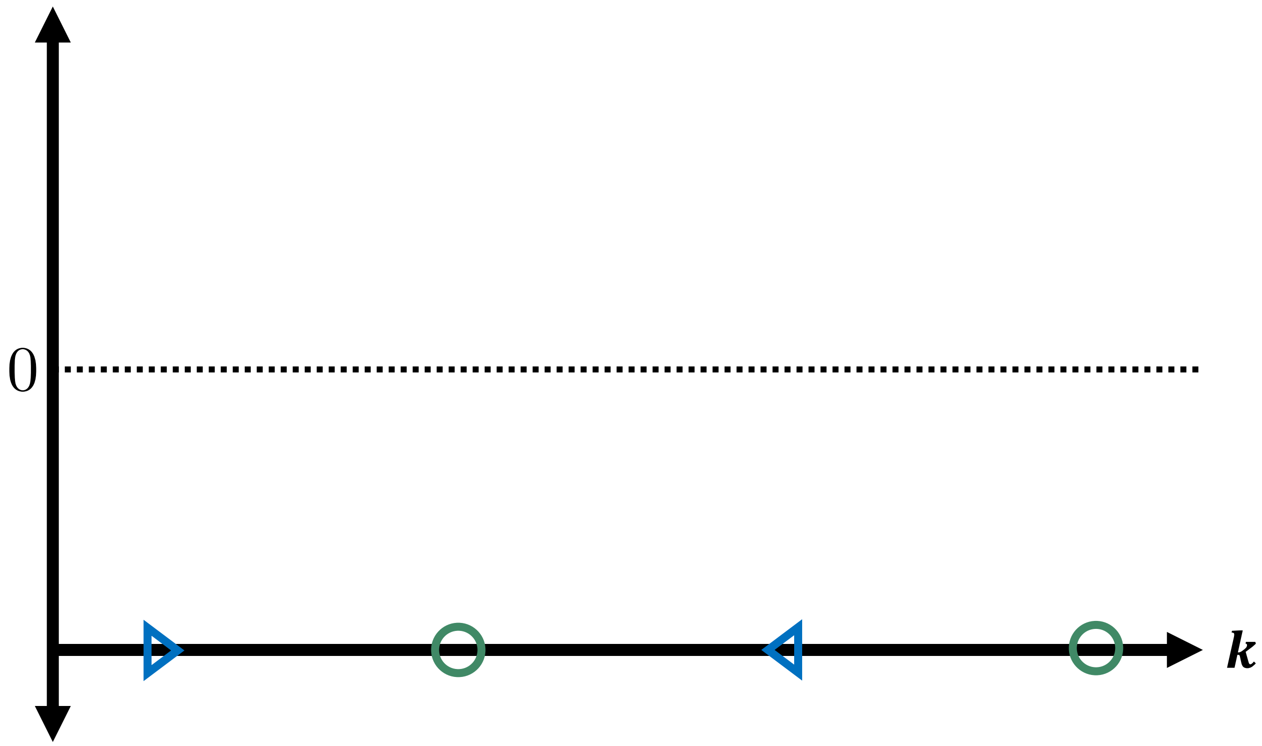


- $\mathcal{S}$: Service Window

$$\mathcal{S} \subseteq \mathcal{T}$$

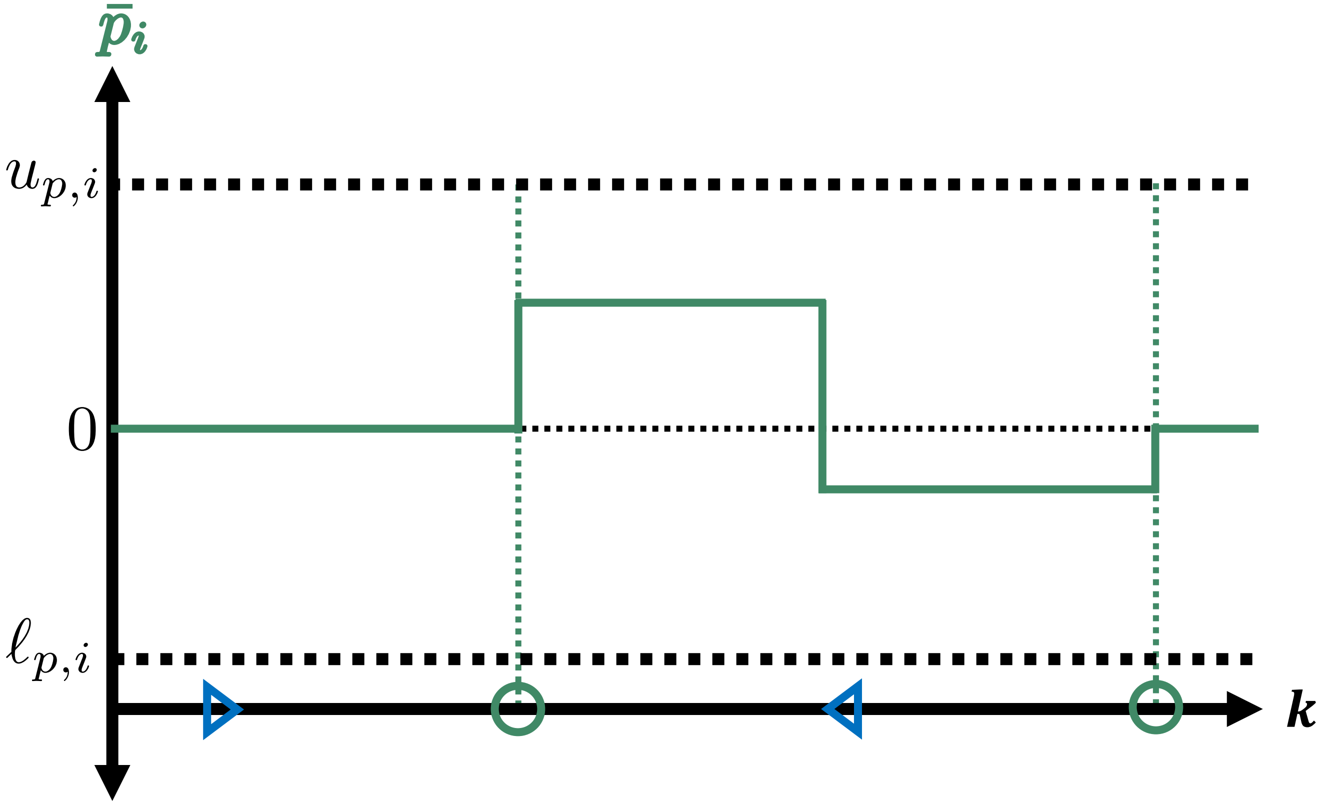
$$\delta(k) = 0, k \in \mathcal{S}^{\mathrm{c}} = \mathcal{T} \setminus \mathcal{S}$$

Problem formulation



- $\mathcal{S}$: Service Window
- $\mathcal{C}_i$: Connection Window

Problem formulation



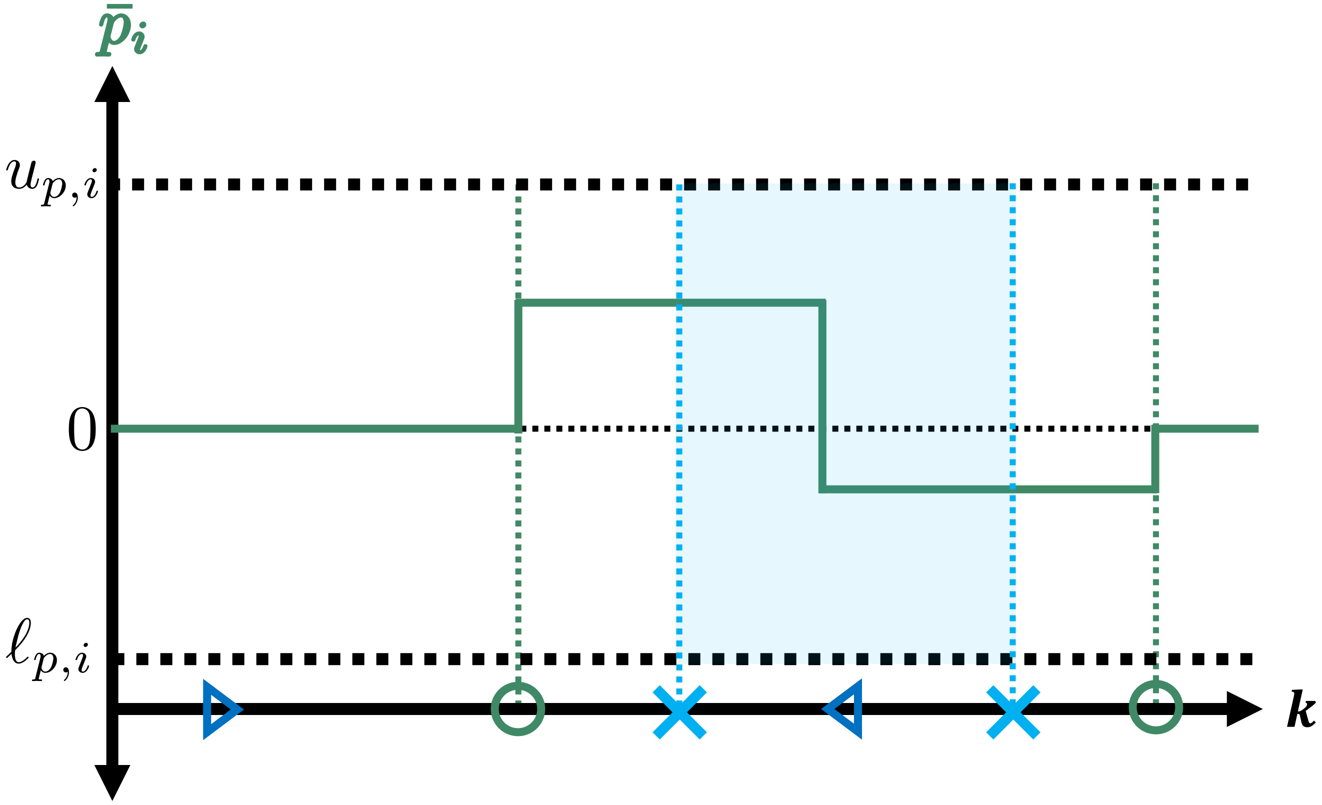
$\mathcal{S}$: Service Window

$\mathcal{C}_i$: Connection Window

$$\bar{p}_i = a_i + g_i$$

$$g_i(k) = a_i(k) = 0, \quad k \in \mathcal{C}_i^c = \mathcal{T} \setminus \mathcal{C}_i$$

Problem formulation



▷ $\mathcal{S}$: Service Window

○ $\mathcal{C}_i$: Connection Window

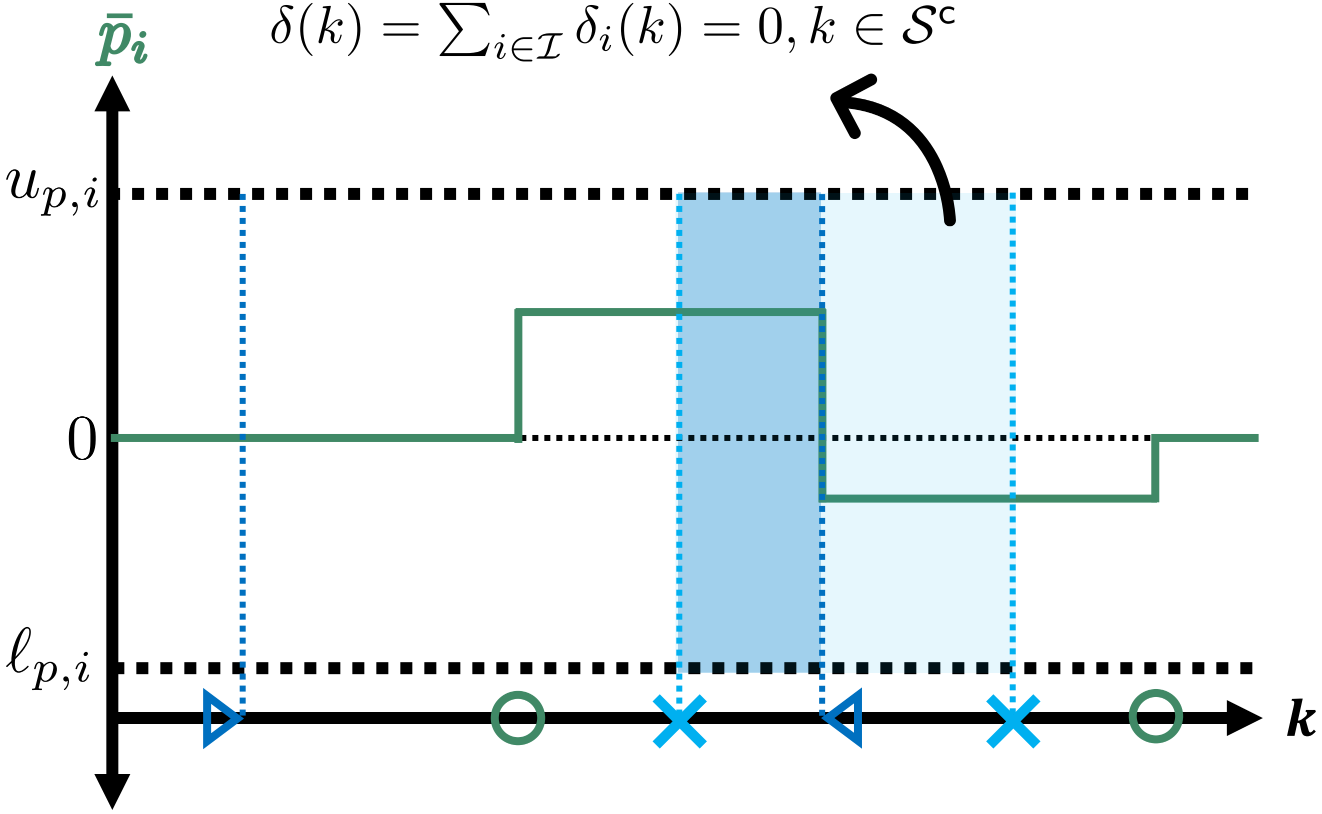
✕ $\mathcal{F}_i$: Flexibility Window

$$\mathcal{F}_i \subseteq \mathcal{C}_i$$

$$\delta_i(k) = 0, k \in \mathcal{F}_i^{c} = \mathcal{T} \setminus \mathcal{F}_i$$

Problem formulation

$$\delta(k) = \sum_{i \in \mathcal{I}} \delta_i(k) = 0, k \in \mathcal{S}^{\mathsf{c}}$$



- $\mathcal{S}$: Service Window
- $\mathcal{C}_i$: Connection Window
- $\mathcal{F}_i$: Flexibility Window

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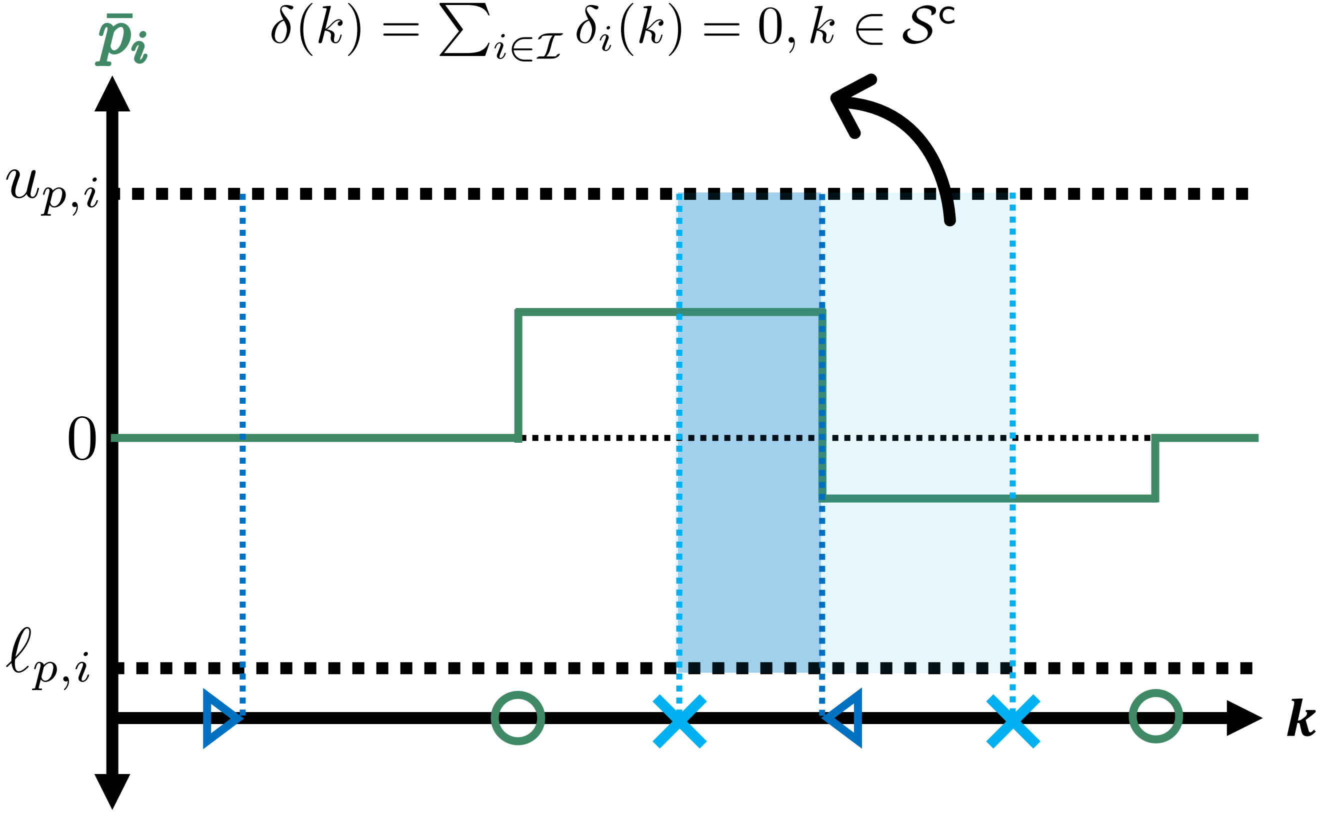

▷ $\mathcal{S}$: Service Window
○ $\mathcal{C}_i$: Connection Window
✕ $\mathcal{F}_i$: Flexibility Window

$\mathcal{F}_i \subseteq \mathcal{C}_i$

$\delta_i(k) = 0, k \in \mathcal{F}_i^c = \mathcal{T} \setminus \mathcal{F}_i$

* for ease of explanation, from now on the time windows are not considered

Flexibility assessment and policy design

We inner approximate the real flexibility Δ using a box:

$$\mathcal{B}_{c,d} = \{\delta \in \mathbb{R}^M : \ |\delta - c| \leq d\} \subseteq \Delta$$

$$d > 0$$

Flexibility assessment and policy design

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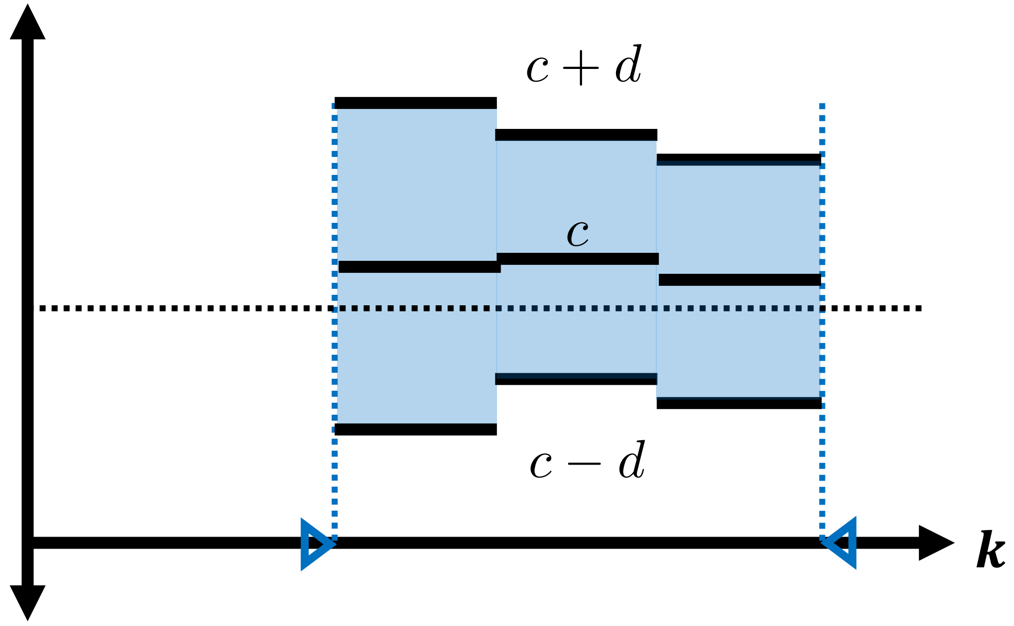

Flexibility assessment and policy design

Parametrize the individual deviation as a linear function of the grid request:

$$\delta_i = K_i \delta, \qquad K_i \in \mathbb{R}^{M \times M}$$

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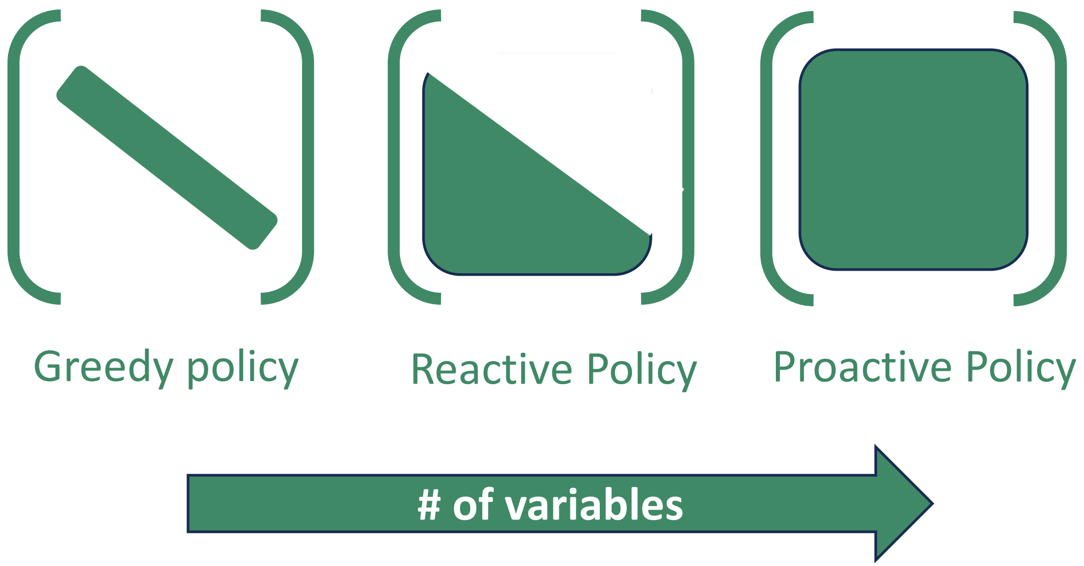
The sparsity pattern of K_i accounts for the information structure.

Sparsity pattern of gain matrix K_i



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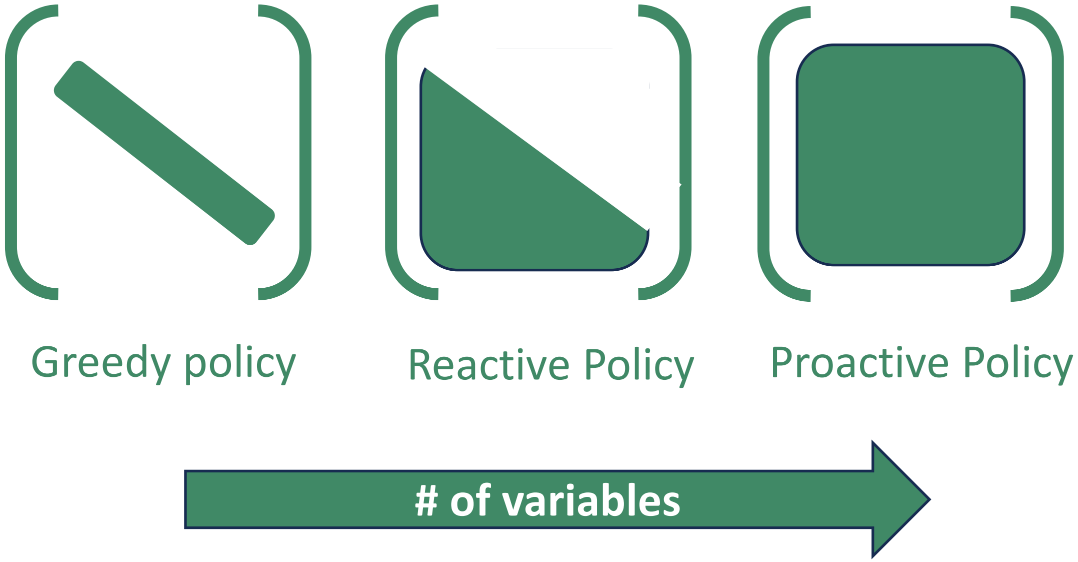

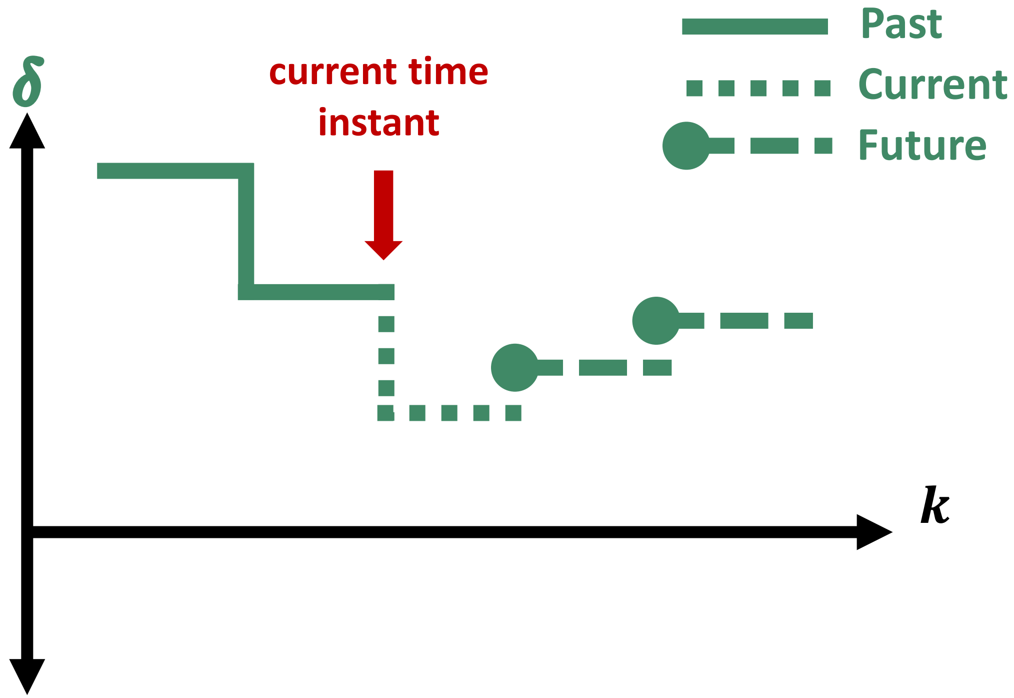

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$$\textstyle\sum_{i \in \mathcal{I}} \delta_i = \delta = \sum_{i \in \mathcal{I}} K_i \delta$$

$$\textstyle\sum_{i \in \mathcal{I}} K_i = I$$

Flexibility assessment and policy design

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$$\begin{aligned} p_i &= \bar{p}_i + K_i \delta \\ &= g_i + a_i + K_i \delta \end{aligned}$$

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$$\Delta = \{\delta \in \mathbb{R}^M : \ \delta = \sum_{i \in \mathcal{I}} \delta_i \wedge (\bar{p} + \delta_i \in \mathcal{P}_i, \ i \in \mathcal{I})\}$$

$$\Delta_K = \{\delta \in \mathbb{R}^M : \ \delta = \sum_{i \in \mathcal{I}} K_i \delta \wedge (\bar{p}_i + K_i \delta \in \mathcal{P}_i, \ i \in \mathcal{I})\}$$

Flexibility assessment and policy design

$$\begin{aligned} \max_{c,d,\{g_i,a_i,K_i\}_{i\in\mathcal{I}}} \quad & f(c,d,g_1,\dots,g_N) \\ \text{subject to:} \quad & \textstyle\sum_{i\in\mathcal{I}} a_i = 0 \\ & \textstyle\sum_{i\in\mathcal{I}} K_i = I \\ & \mathcal{B}_{c,d} \subseteq \Delta_K \end{aligned}$$

$$\bar{p}_i = g_i + a_i$$

$$\Delta_K = \{\delta \in \mathbb{R}^M : \ \delta = \textstyle\sum_{i\in\mathcal{I}} K_i\delta \wedge (\bar{p}_i + K_i\delta \in \mathcal{P}_i, \ i \in \mathcal{I})\}$$

Flexibility assessment and policy design

$$
\begin{array}{ll}
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\text{subject to:} & \sum_{i\in\mathcal{I}} a_i = 0 \\
& \sum_{i\in\mathcal{I}} K_i = I \\
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Cost Function

Geometric-based:

$$\log\big(\text{vol}(\mathcal{B}_{c,d})\big) = \sum_k \log(2\,d(k))$$

$$\sum_k 2\,d(k)$$

Flexibility assessment and policy design

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$$\log\big(\mathrm{vol}(\mathcal{B}_{c,d})\big) = \sum_k \log(2\,d(k))$$

$$\sum_k 2\,d(k)$$

Revenue-based:

$$\sum_k 2\,\rho_k d(k)$$

$$\sum_k \rho_k^+(c(k)+d(k)) - \rho_k^-(c(k)-d(k))$$

Weighted combination

Flexibility assessment and policy design

$$
\begin{aligned}
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Flexibility assessment and policy design

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& F_i(g_i + a_i + K_i c) + |F_i K_i| d \le h_i \quad i \in \mathcal{I}
\end{aligned}
$$

→ Disaggregation feasibility

Flexibility assessment and policy design

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→ Disaggregation feasibility

Flexibility assessment and policy design

By the following change of variable:

$$\gamma_i = a_i + K_i c$$

$$G_i = K_i \mathrm{diag}(d)$$

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if f is convex, the resulting problem is convex

Constraint-coupled structure

$$\max_{c,d,\bar{p},\{\gamma_i,G_i,g_i\}_{i\in\mathcal{I}}} \quad f_{N+1}(c,d,\bar{p}) + \sum_{i=1}^{N} f_i(g_i)$$

subject to: **1. Coupling:**

$$c = \textstyle\sum_{i\in\mathcal{I}} \gamma_i,\ \operatorname{diag}(d) = \sum_{i\in\mathcal{I}} G_i,\ \bar{p} = \sum_{i\in\mathcal{I}} g_i,$$

2. Aggregator $(i = N+1)$:

$$d > 0,$$

3. Prosumers:

$$F_i(g_i + \gamma_i) + |F_i G_i| 1_M \leq h_i, \quad i \in \mathcal{I}.$$

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$$F_i(g_i + \gamma_i) + |F_i G_i| 1_M \leq h_i, \quad i \in \mathcal{I}.$$

local decision variables $\quad x_i = (G_i, \gamma_i, g_i) \quad i = 1, \ldots, N, \qquad x_{N+1} = (d, c, \bar{p})$

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local constraints $\mathcal{X}_i$

local decision variables $x_i = (G_i, \gamma_i, g_i) \quad i = 1, \ldots, N$, $\quad x_{N+1} = (d, c, \bar{p})$

Constraint-coupled structure

separable cost

$$\max_{c,d,\bar{p},\{\gamma_i,G_i,g_i\}_{i\in\mathcal{I}}} \quad f_{N+1}(c,d,\bar{p}) + \sum_{i=1}^{N} f_i(g_i)$$

subject to: **1. Coupling:**

$$c = \textstyle\sum_{i\in\mathcal{I}} \gamma_i, \;\; \mathrm{diag}(d) = \sum_{i\in\mathcal{I}} G_i, \;\; \bar{p} = \sum_{i\in\mathcal{I}} g_i,$$

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Constraint-coupled structure

$$\min_{\{x_i\}_{i=1}^{N+1}} \sum_{i=1}^{N+1} f_i(x_i)$$

$$\text{subject to:} \quad \sum_{i=1}^{N+1} A_i x_i = b, \; b \in \mathbb{R}^{n_c}, \; A_i \in \mathbb{R}^{n_c \times n_{a,i}},$$

$$x_i \in \mathcal{X}_i, \quad \forall i = 1, ..., N+1.$$

Constraint-coupled structure

$$\sum_{i=1}^{N+1} A_i x_i = \begin{bmatrix} \sum_{i\in\mathcal{I}} \gamma_i - c \\ \sum_{i\in\mathcal{I}} G_i - \mathrm{diag}(d) \\ \sum_{i\in\mathcal{I}} g_i - \bar{p} \end{bmatrix}$$

$$b = 0$$

$$\min_{\{x_i\}_{i=1}^{N+1}} \quad \sum_{i=1}^{N+1} f_i(x_i)$$

$$\text{subject to:} \quad \sum_{i=1}^{N+1} A_i x_i = b, \; b \in \mathbb{R}^{n_c}, \; A_i \in \mathbb{R}^{n_c \times n_{a,i}},$$

$$x_i \in \mathcal{X}_i, \quad \forall i = 1, \ldots, N+1.$$

ADMM algorithm

Initialization:

$x_{i,0} \in \mathcal{X}_i, \;\; i = 1, \ldots, N+1.$

$z_0 = \frac{1}{N+1}\left(\sum_{i=1}^{N+1} A_i x_0 - b\right)$

$\lambda_0 \in \mathbb{R}^{n_c}$

Repeat until convergence:

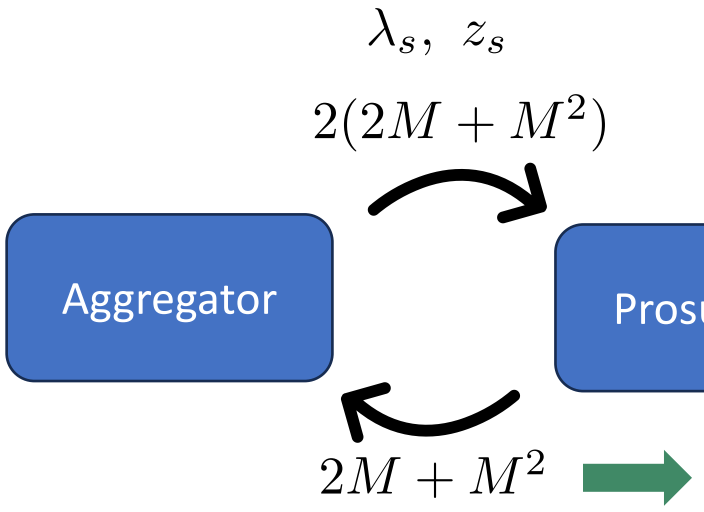
$x_{i,s+1} \in \arg\min_{x_i \in \mathcal{X}_i} \left\{ f_i(x_i) + \lambda_s^\top A_i x_i + \frac{\alpha}{2}\|A_i x_i - A_i x_{i,s} + z_s\|^2 \right\}$

$z_{s+1} = \frac{1}{N+1}\left(\sum_{i=1}^{N+1} A_i x_{i,s+1} - b\right)$

$\lambda_{s+1} = \lambda_s + \alpha\, z_{s+1}$

$s \leftarrow s + 1$

Communication effort



Vector containing the violation of coupling constraint

Memory requirements

$$\begin{aligned} \max_{c,d,\{g_i,\gamma_i,G_i\}_{i\in\mathcal{I}}} \quad & f(c,d,g_1,\ldots,g_N) \\ \text{subject to:} \quad & c = \textstyle\sum_{i\in\mathcal{I}} \gamma_i \\ & \mathrm{diag}(d) = \textstyle\sum_{i\in\mathcal{I}} G_i \\ & F_i(g_i+\gamma_i) + |F_i G_i| 1_M \le h_i \quad i\in\mathcal{I} \end{aligned}$$

Number of variables centralized:

$$2M + N(2M + M^2)$$

Memory requirements

Single Prosumer

A single prosumer manage: $\gamma_i, g_i, G_i \Rightarrow 2M + M^2$

Using ADMM algorithm introduce two variable vectors with size equal to the coupling constraints. $\lambda_s,\ z_s \Rightarrow 2(2M + M^2)$

Number of variables prosumers (memory):

$$6M + 3M^2$$

Aggregator

Aggregator treated as an additional prosumer: $c, d, \bar{p} \Rightarrow 3M$

Using ADMM algorithm introduce two variable vectors with size equal to the coupling constraints. $\lambda_s,\ z_s \Rightarrow 2(2M + M^2)$

Number of variables aggregator (memory):

$$7M + 2M^2$$

Decentralized versus centralized

$\mathcal{T} = \{1, \ldots, 4\}, \; \tau = 15$ min

half ACs and half RFs

N	Iterations	Dec. time [min]	Cen. time [min]
20	171	2.303	0.033
30	208	2.843	0.058
40	240	3.240	0.069
50	267	3.516	0.137
100	347	4.696	0.312
200	361	4.934	1.184
300	337	4.437	2.162
400	381	5.080	3.330
500	369	4.859	5.911
600	406	5.549	12.71

Main contribution

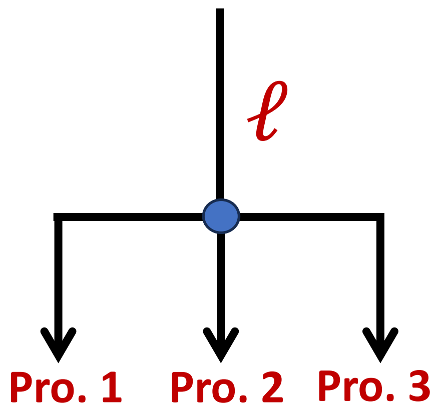
We propose a flexible and comprehensive 1-BP method accounting for:

1. Box-shaped flexibility set & disaggregation policy.
2. Joint optimization of the baseline profile.
3. Inclusion of distribution network constraints
4. Possibility of parallelizing the computation while addressing network constraints.
5. Enhancing the aggregation of multiple prosumers by:
   - preserving the privacy of local information.
   - including loads with limited time availability.

Additional constraints

Minimum flexibility:

$$c + d \geq u_{\min}, \quad c - d \leq \ell_{\min}, \quad \Rightarrow \quad \mathbf{0} \subseteq \mathcal{B}_{c,d}$$



Network Constraints:

$$\ell_n^{(j)} \leq \min_{\delta \in \mathcal{B}_{c,d}} \sum_{i \in \mathcal{N}_j} p_i \leq \max_{\delta \in \mathcal{B}_{c,d}} \sum_{i \in \mathcal{N}_j} p_i \leq u_n^{(j)}$$

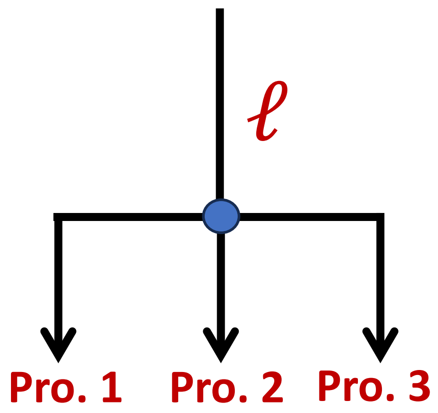
$$\mathcal{N}_j \subseteq \mathcal{I}$$

Additional constraints

Minimum flexibility:

$$c + d \geq u_{\min},$$
$$c - d \leq \ell_{\min},$$

$\Rightarrow$ $\mathbf{0} \subseteq \mathcal{B}_{c,d}$



Network Constraints:

$$\sum_{i \in \mathcal{N}_j} (g_i + \gamma_i) + \left|\sum_{i \in \mathcal{N}_j} G_i\right| 1_M \leq u_n^{(j)}$$
$$\sum_{i \in \mathcal{N}_j} (g_i + \gamma_i) - \left|\sum_{i \in \mathcal{N}_j} G_i\right| 1_M \geq \ell_n^{(j)}$$

Simulation examples

TCL aggregation

$$\mathcal{T} = \{1, 2\}$$
$$N = 2$$

Maximize the volume using:

- Greedy Policy (red)
- Reactive Policy (blue)

Compare to

- Approximation given by Zhao (yellow)

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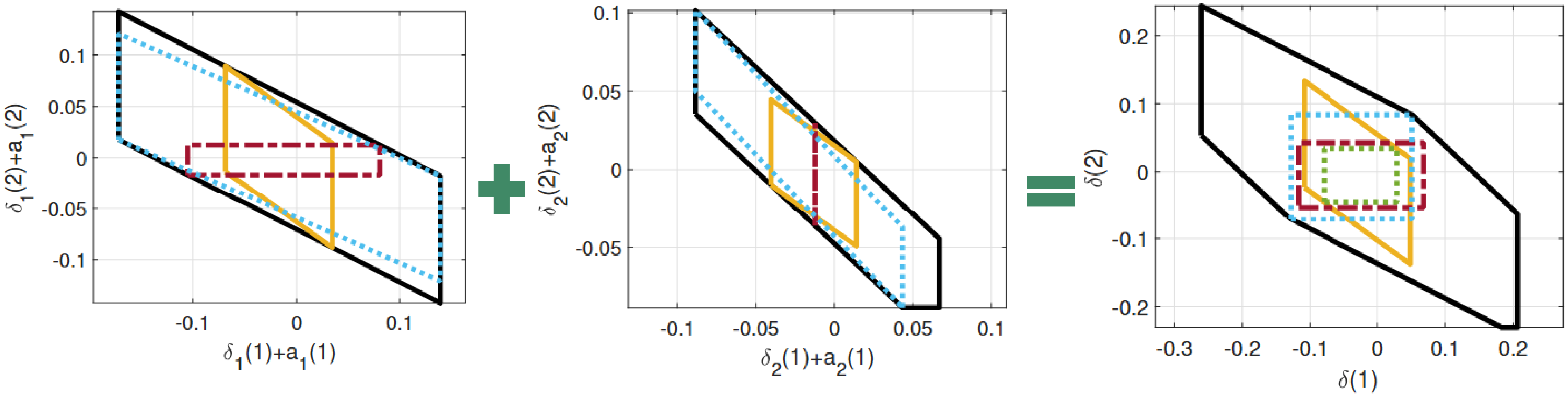


(a) AC 1 space (b) AC 2 space (c) Aggregated space

TCL aggregation

$$\mathcal{T} = \{1, \ldots, 24\}, \; \tau = 15 \text{ min}$$

$N = 100$ Residential ACs

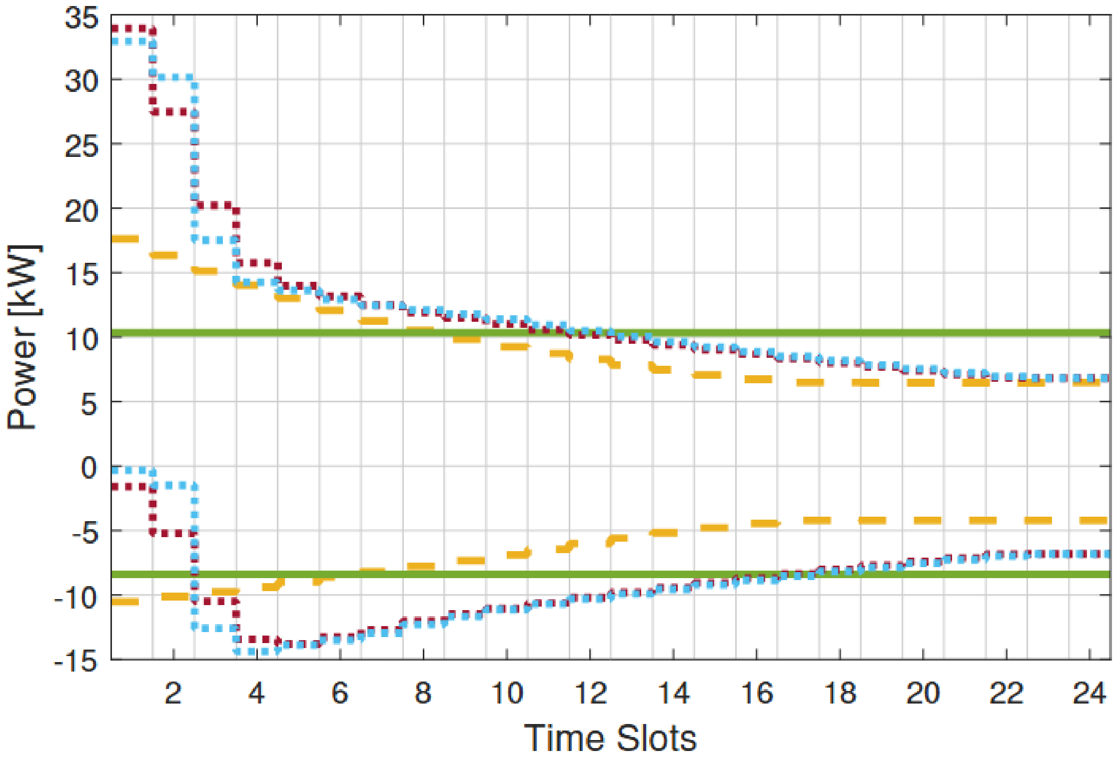
Maximize the volume using:

- Constant Policy (green)
- Greedy Policy (red)
- Reactive Policy (blue)

Compare with

- Maximum volume box inside approximation given by Zhao (yellow)

We compute the **Volume Ratio** between "our" boxes and the maximum box of Zhao



Policy	Volume ratio
Constant	2.13×10^2
Greedy	15.93×10^2
Reactive	18.74×10^2

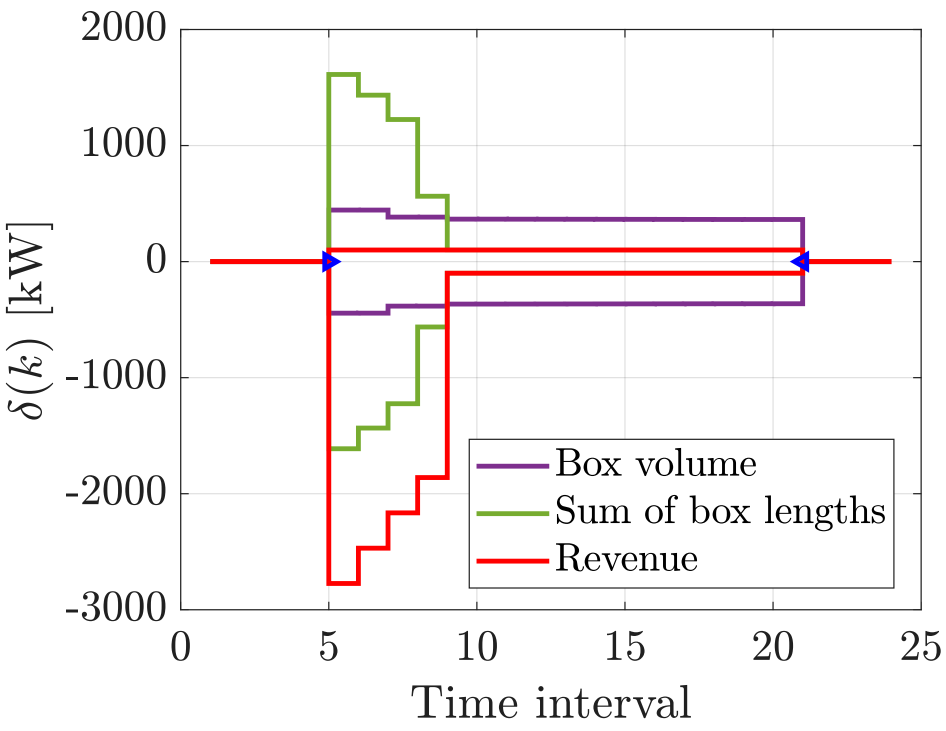
Aggregation of heterogeneous units

$\mathcal{T} = \{1, \ldots, 24\},\ \tau = 15\ \text{min}$

$N = 100$

20 units of each type of the following units: ACs, RFs, EVs, EBs, and PGs.

The pool is offering balancing services in a 4-hour service window S = {5,...,20}, with minimum required upward and downward flexibility of 100 kW



Cost function	Volume [kW^{16}]	Power [kW]	Revenue [euros]
Volume	**13,68**	1,61 x 10^3	225,54
Power	13,38	**3,43 x 10^3**	481,04
Revenue	13,26	3,07 x 10^3	**509,77**

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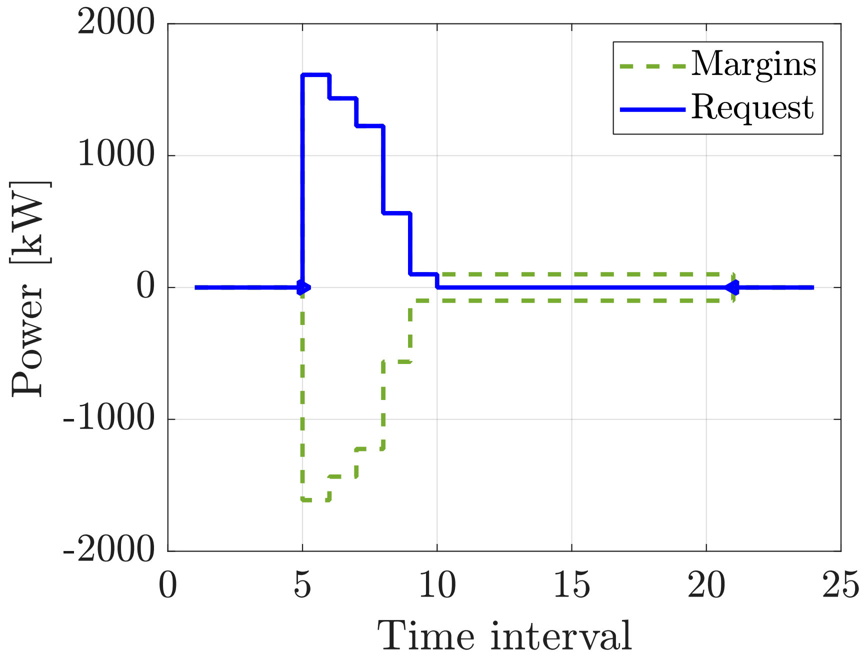

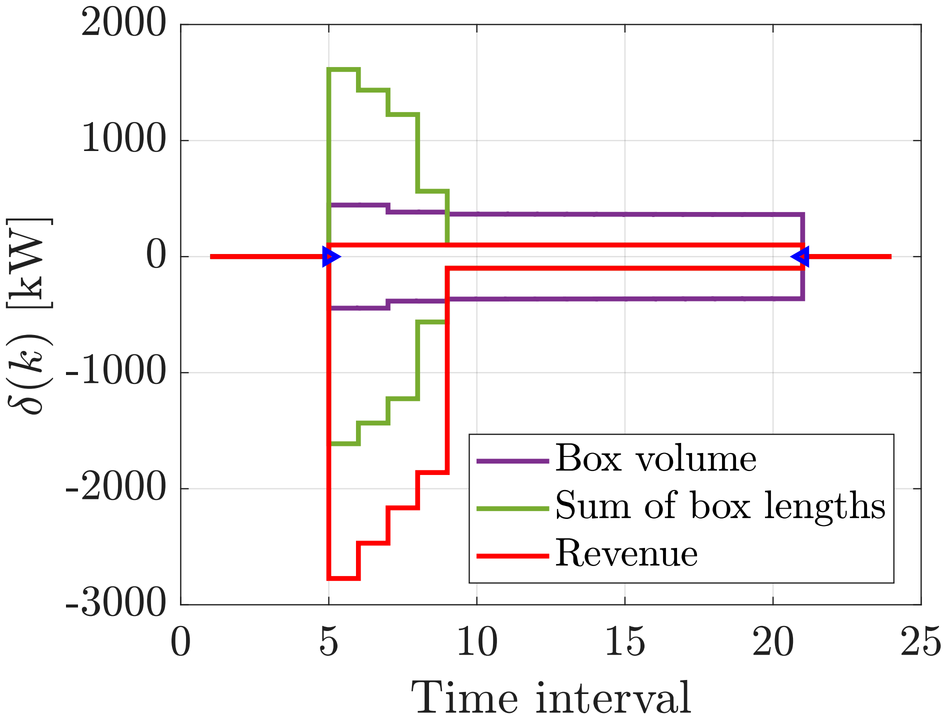

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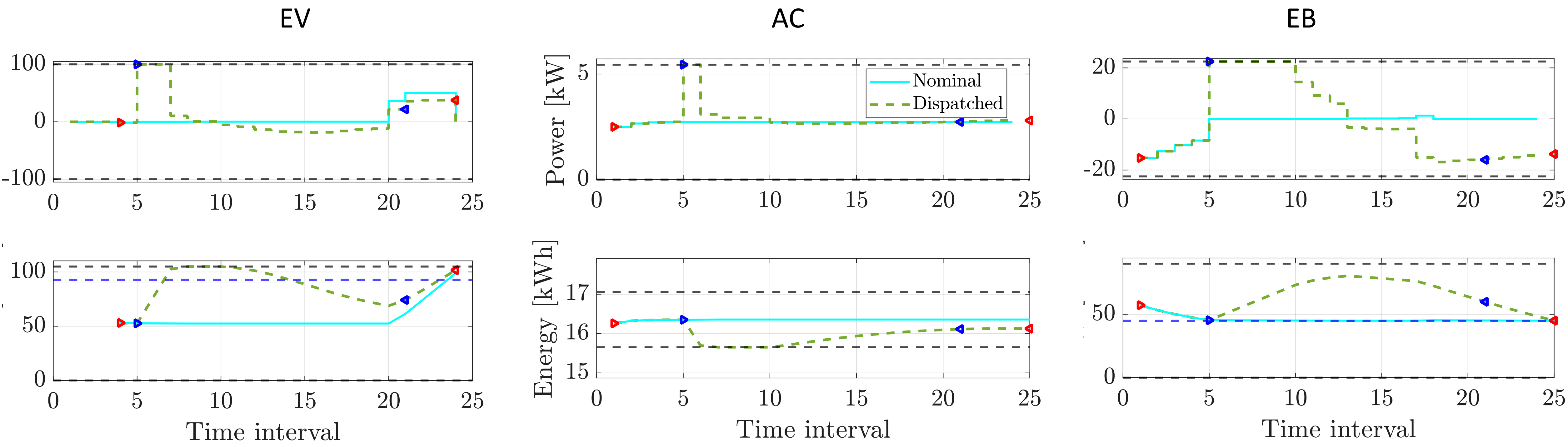
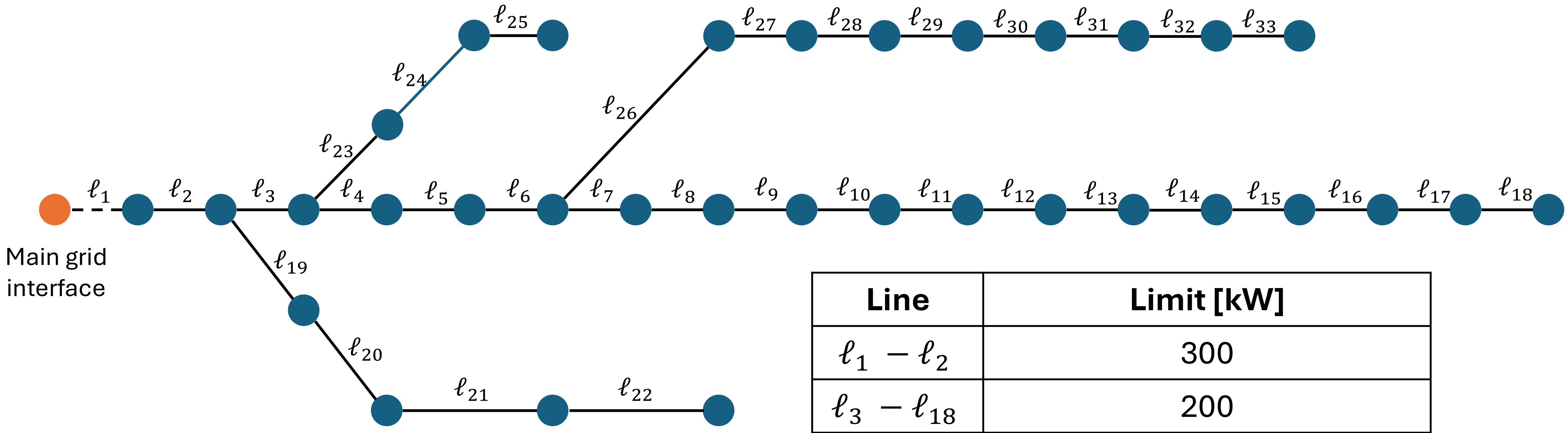


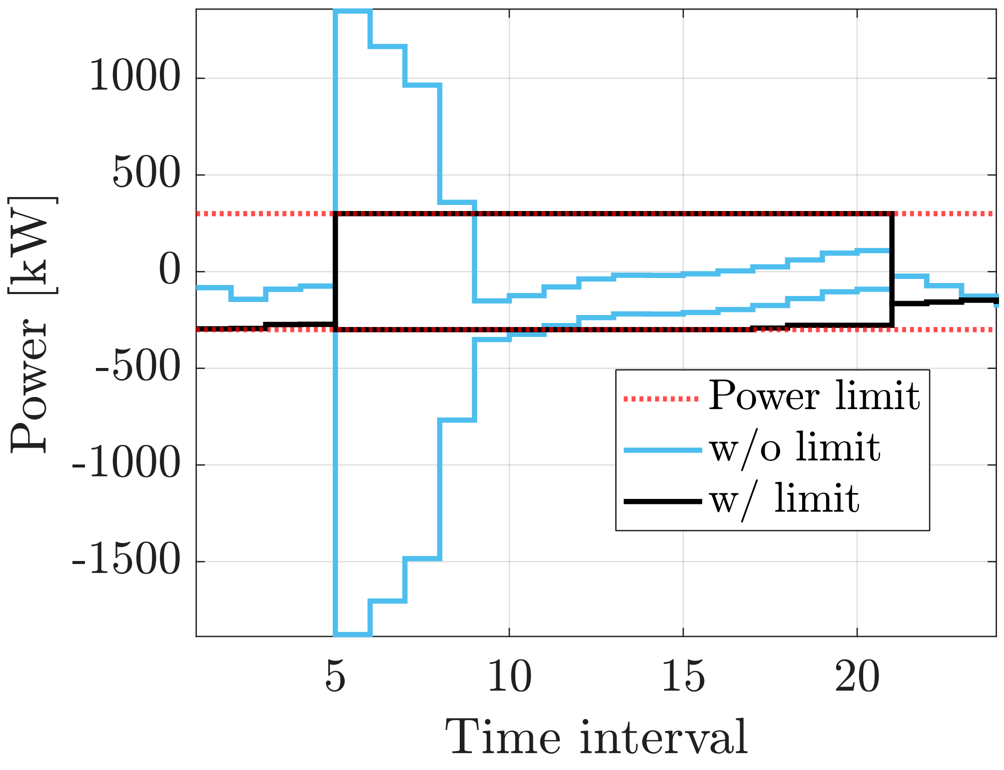
Aggregation of heterogeneous units



Line	Limit [kW]
$\ell_1 - \ell_2$	300
$\ell_3 - \ell_{18}$	200
$\ell_{19} - \ell_{33}$	100

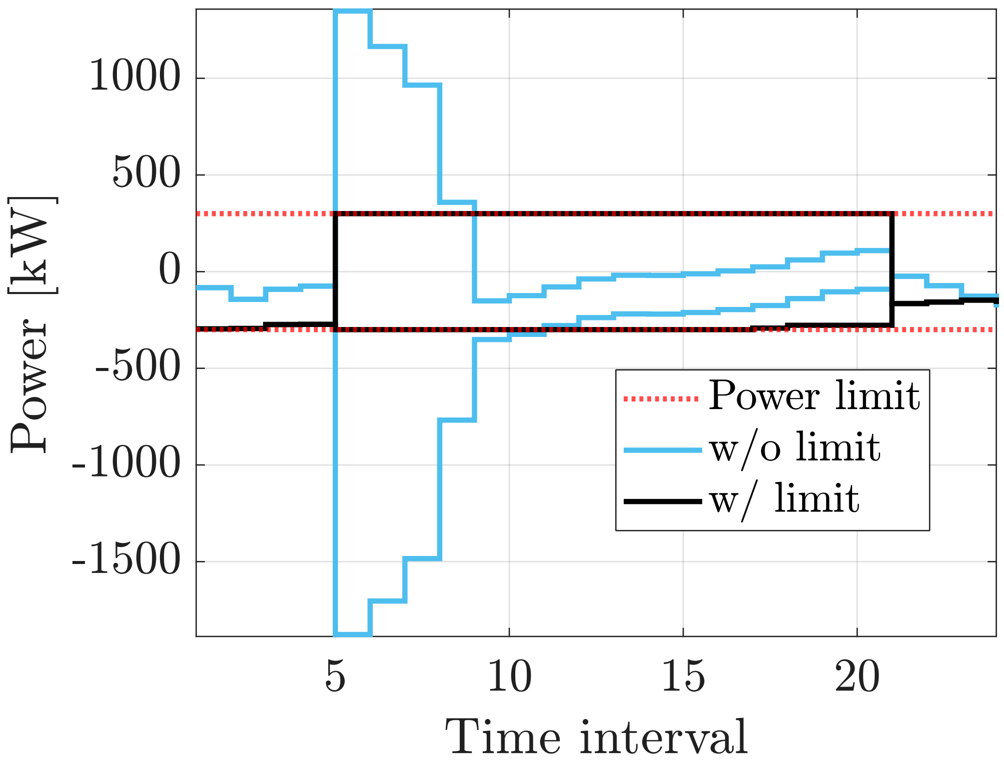
IEEE 33-bus distribution system

Aggregation of heterogeneous units

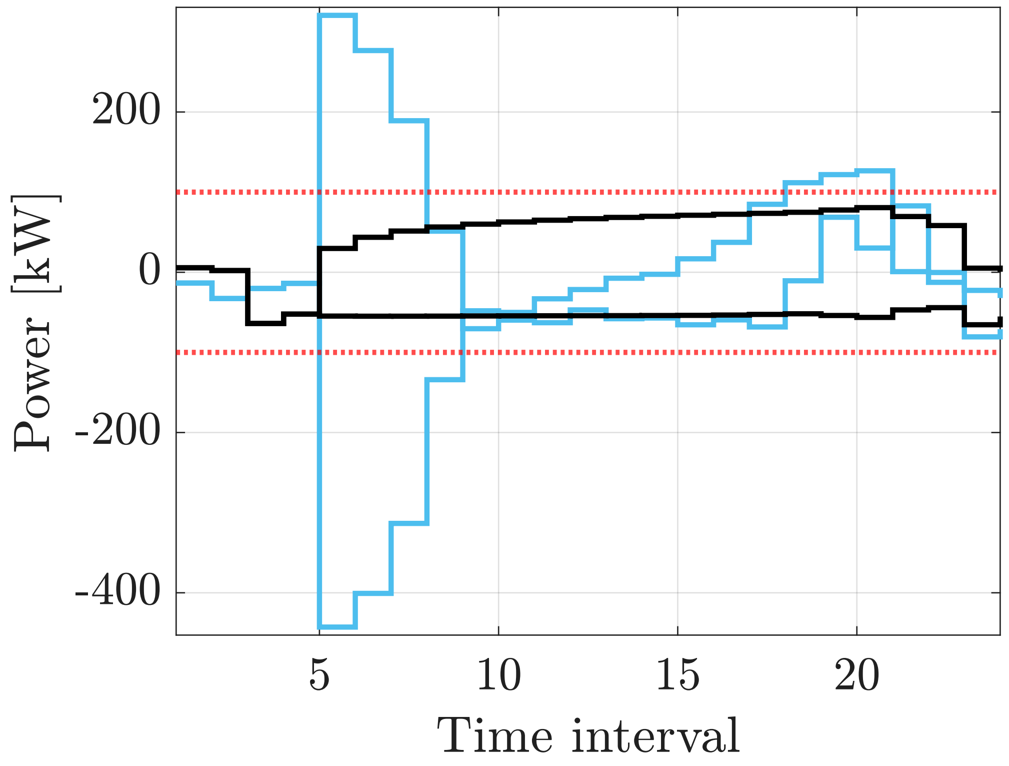


Flexibility provided through line 1
(at the node interfacing with the grid)

Aggregation of heterogeneous units

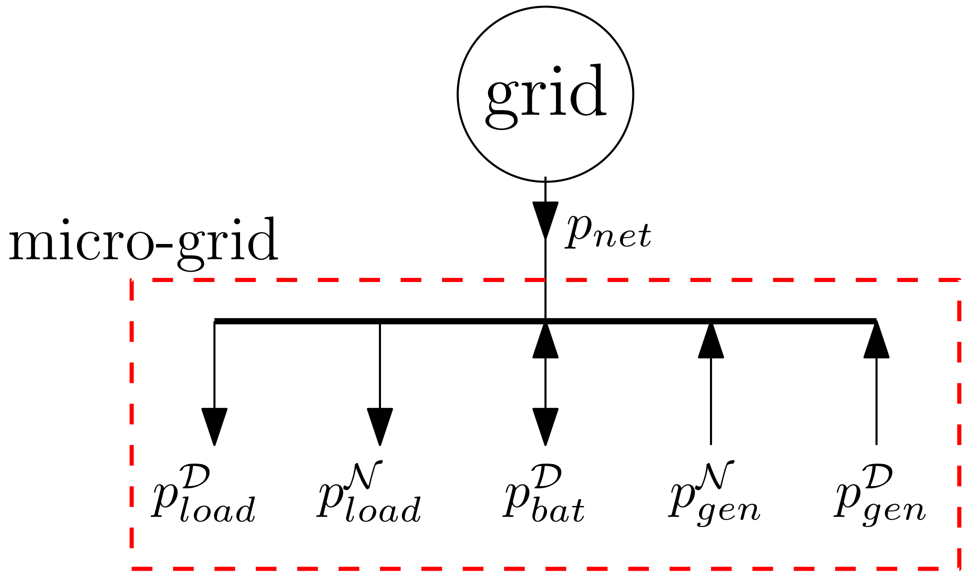


Flexibility provided through line 1
(at the node interfacing with the grid)



Flexibility provided through line 26

Microgrid



Self-consumption objective: net zero exchange with the main grid over one-day horizon
→ additional constraint that the nominal power of dispatchable units should be equal to minus the non-dispatchable net power

Microgrid

Non-dispatchable units

Unit	Min Power (kW)	Max Power (kW)
Resistive Load	0	16
Electronic Load	0	12
PV Generation	0	17

Generators

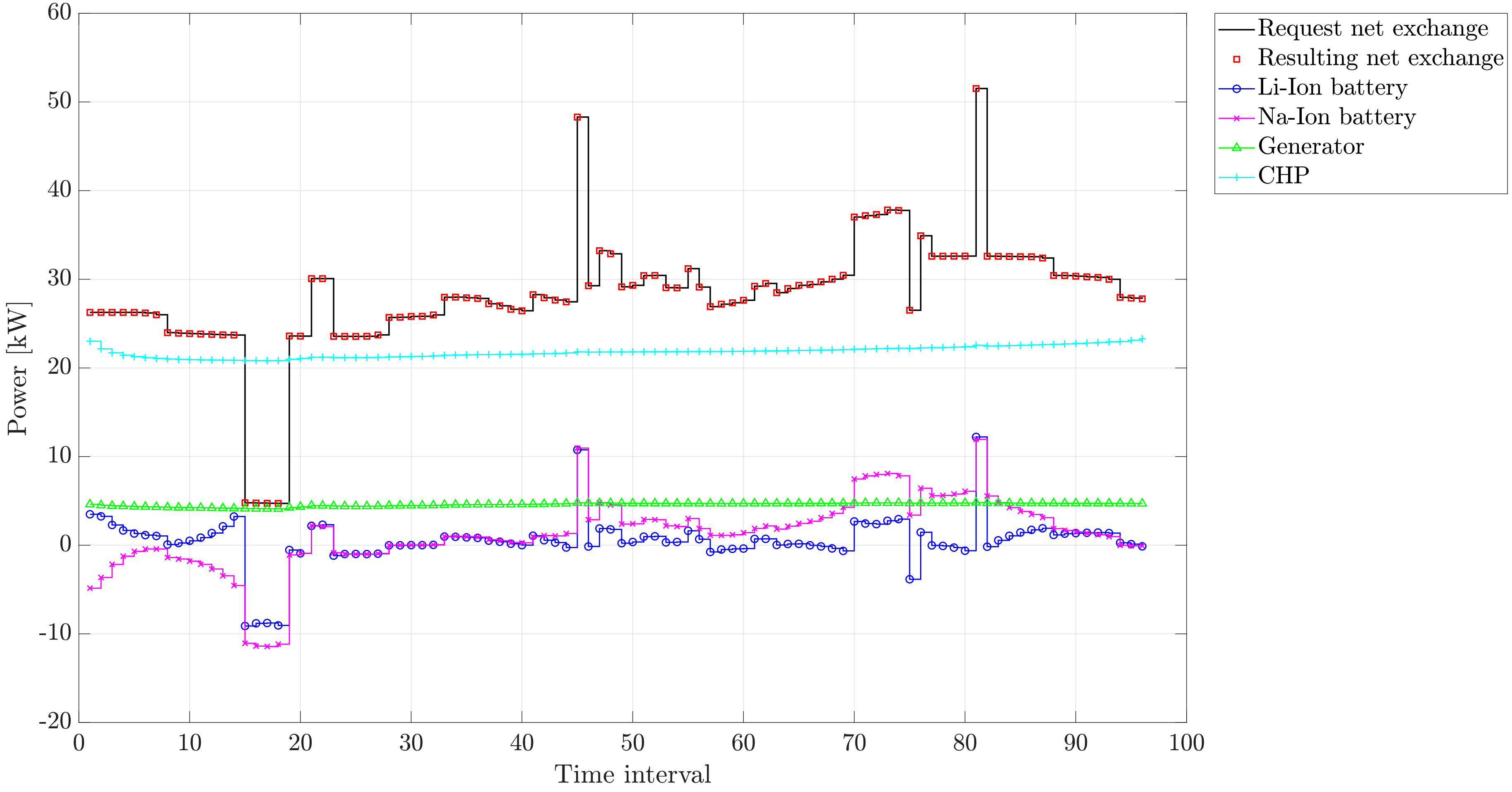
Unit	Min Power (kW)	Max Power (kW)
Small-scale Generator	0	5
Combined Heat and Power (CHP) Generator	5	25

Sodium and lithium batteries

Parameter	Value
Nominal Energy	20 kWh
min/max power	-20 / 20 kW
SoC min/max	10% / 90%
Initial SoC	30%
Self-Discharge	0.999

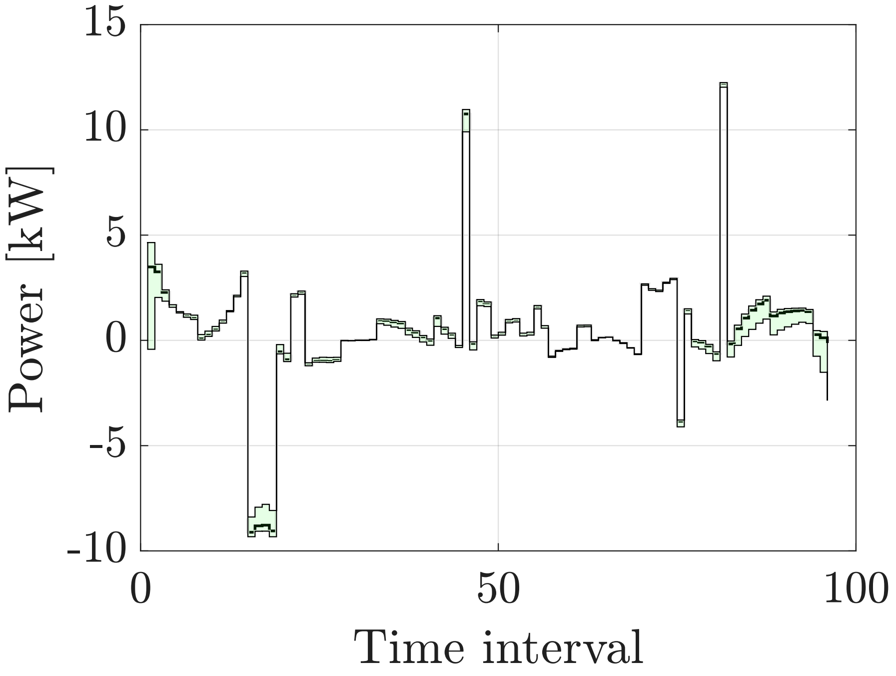
Parameter	Value
Nominal Energy	67 kWh
min/max power	-18 / 18 kW
SoC min/max	10% / 90%
Initial SoC	40%
Self-Discharge	0.999

Microgrid: nominal power profiles

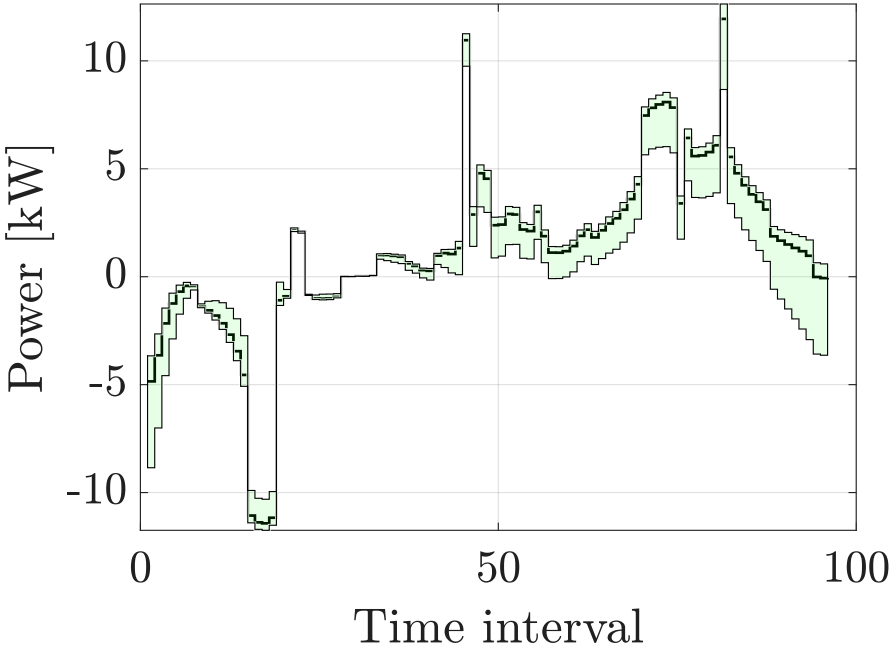


Microgrid: flexibility margins

Li-ion battery

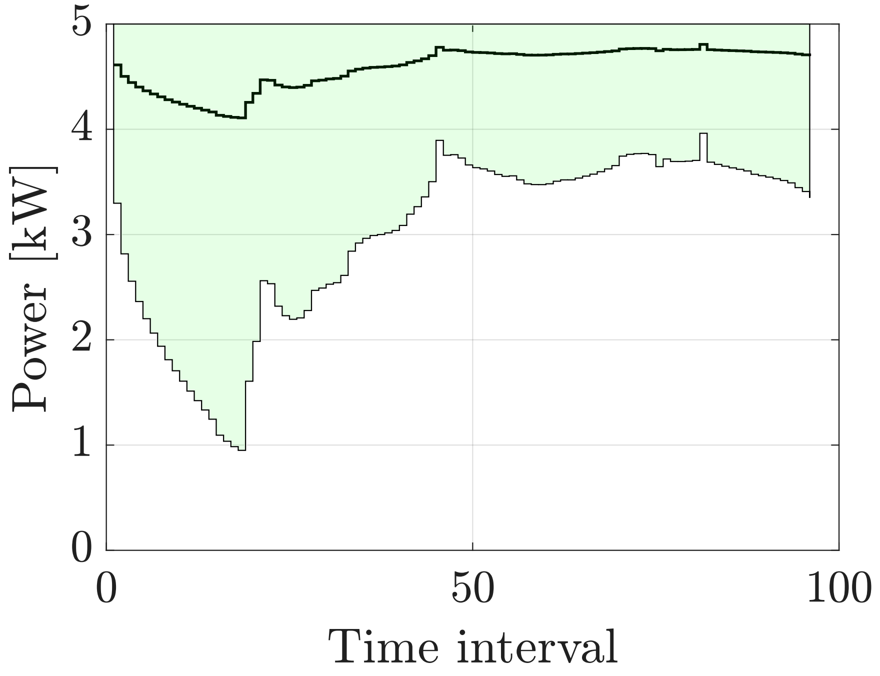


Na-Ion battery

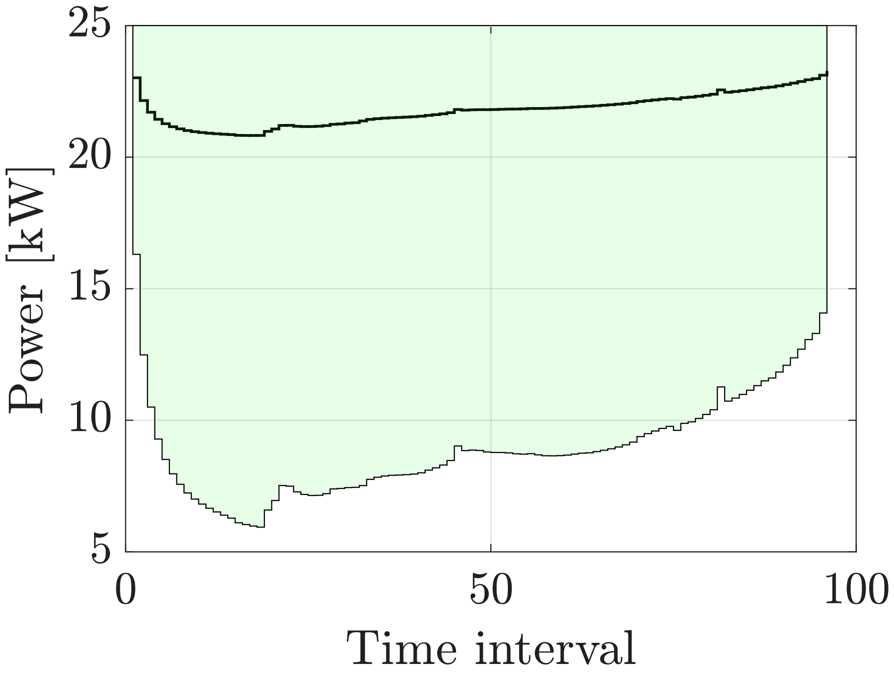


Microgrid: flexibility margins

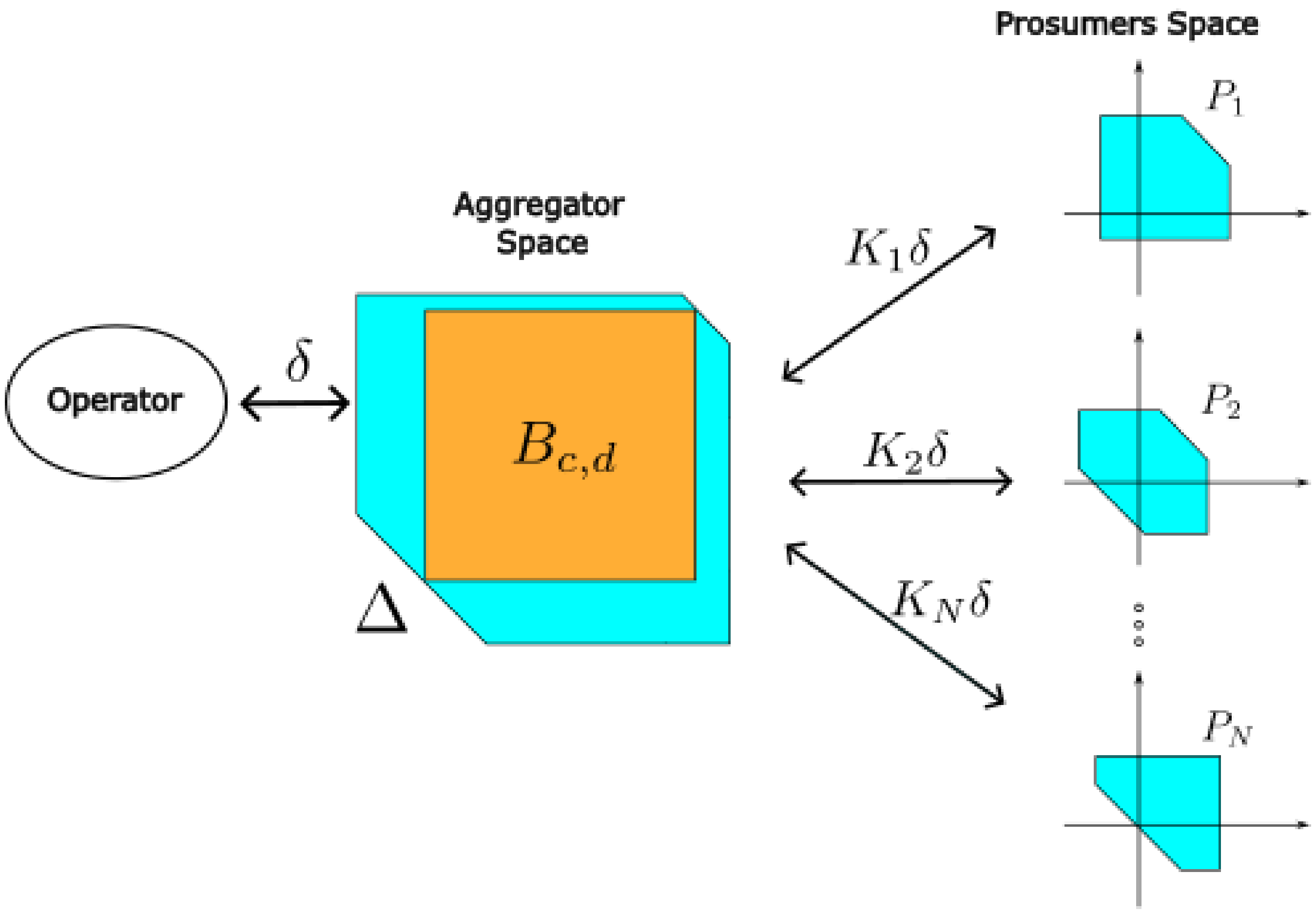
Generator



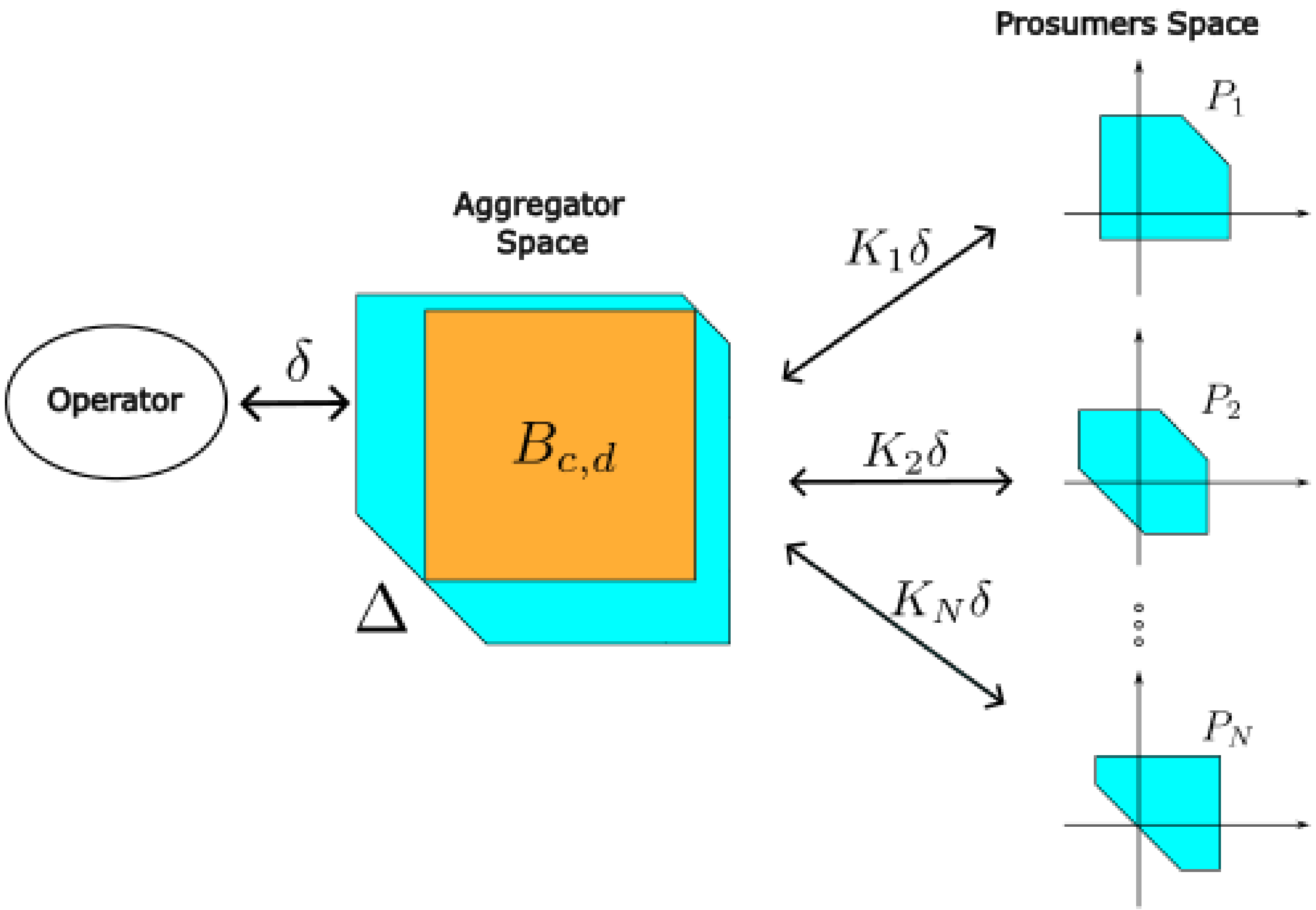
CHP generator



Conclusions and future work



Conclusions and future work



Research plan

- Make a cost assessment (including peer-to-peer and grid exchange cost + operational costs)
- Account for the different prosumers' willingness to offer flexibility when designing the disaggregation policy
- Introduce a policy that is robust with respect to the initial conditions.
- Studying the case of possibly non-convex energy resources.

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