ECODREAM

Energy COmmunity management:

DistRibutEd AlgorithMs and toolboxes for efficient and sustainable operations

Virginia Casella
virginia.casella@edu.unige.it
Luca Parodi
luca.parodi@edu.unige.it
Giulio Ferro
giulio.ferro@edu.unige.it

Introduction

Reduce Green House Gases emissions by at least **55% by 2030** compared to 1990 levels.

Energy transition from an energy mix centered on fossil fuels to one with low or zero carbon emissions.

- Use of renewable resources
- Distributed generation
- Electric mobility
- Prosumers (smart microgrids, smart buildings)

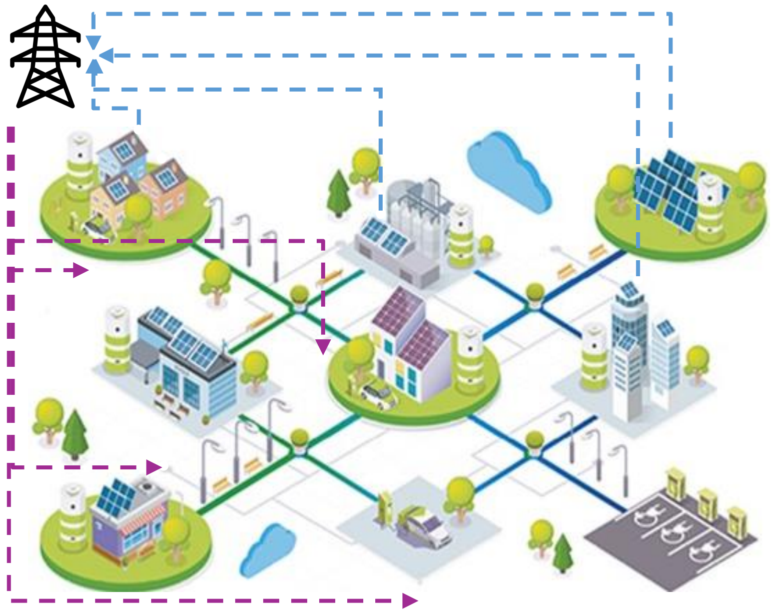
Introduction

Reduce Green House Gases emissions by at least **55% by 2030** compared to 1990 levels.

Energy transition from an energy mix centered on fossil fuels to one with low or zero carbon emissions.

New Energy Management Systems (EMSs) and optimization models to promote **self-consumption**

Renewable Energy Communities



They are a set of at least **two members**: a consumer and a production plant connected under the same primary substation.

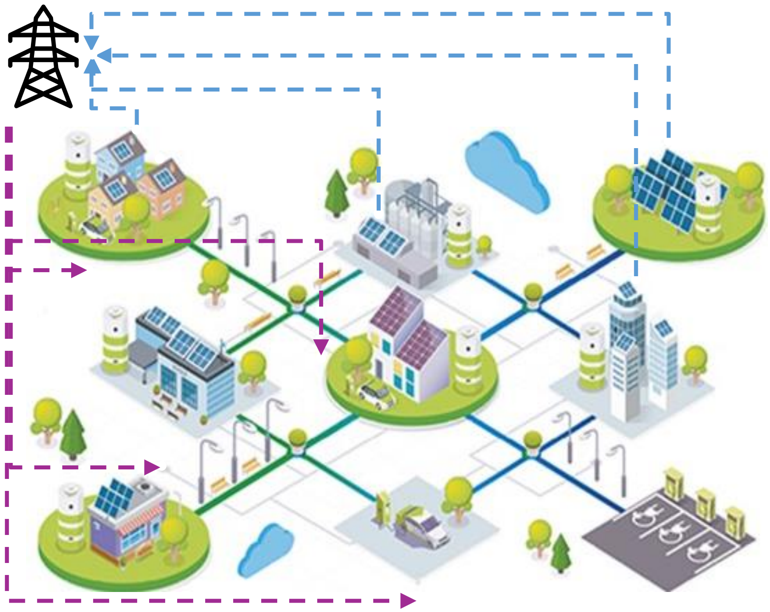
They are an autonomous legal entity with the aim of providing economic, social and environmental benefits.

Participation must be open and voluntary.

The production plants must be exclusively from **renewable sources**.

An **ESS** can be provided, and the unused energy produced can be sold on the grid.

Shared energy



Shared energy: minimum, for each hour, between the electricity injected into the grid by renewable plants and the electricity withdrawn by all end customers associates (also through ESSs).

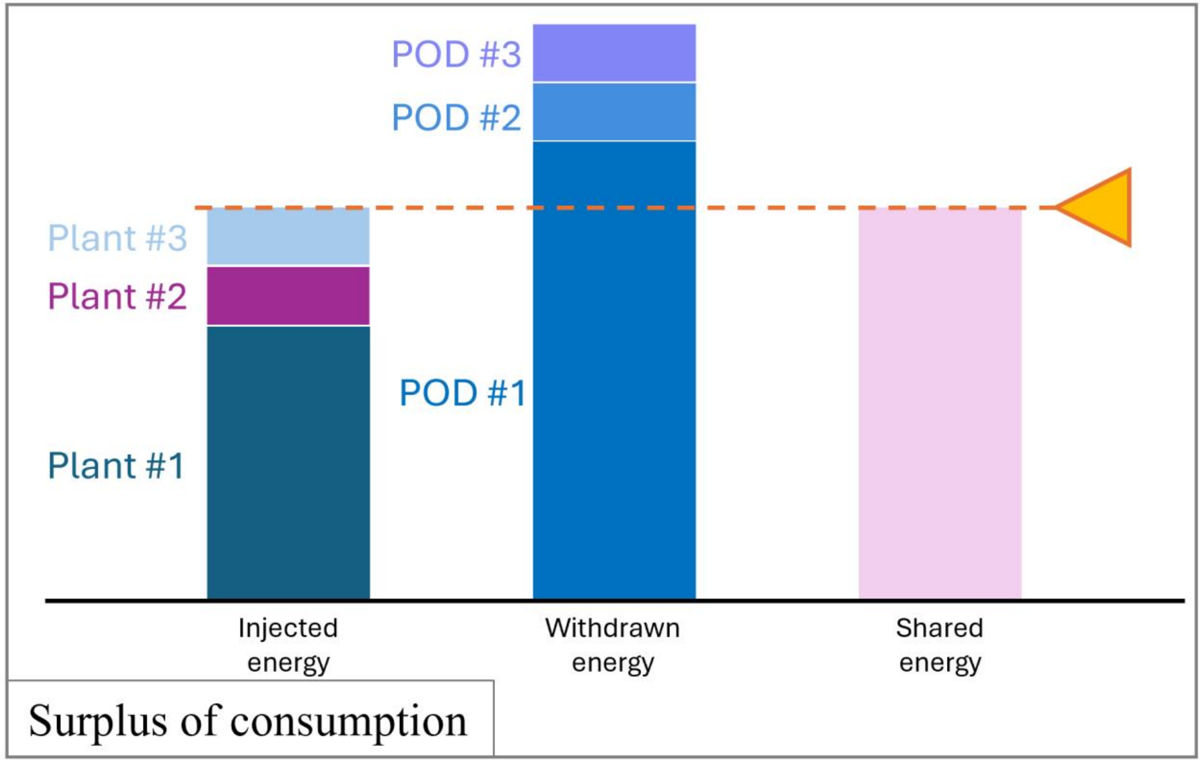
Perimeter: withdrawal and input points must be located on low voltage electrical networks underlying the same secondary substation.

The sharing of the electricity produced occurs using the existing distribution network.

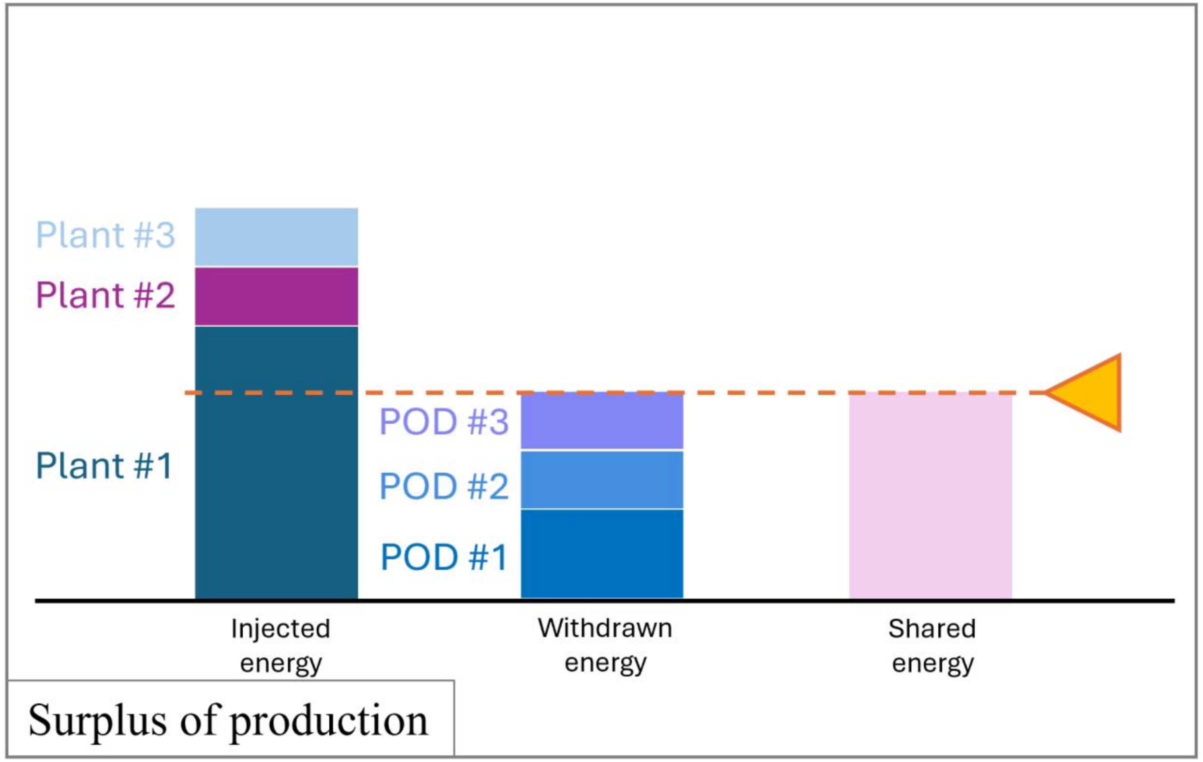
All energy exchanges are **virtual**
→ there is not the modelling of the electrical grid like in microgrids, sustainable energy districts and smart grids.

Shared energy

Shared energy: minimum, for each hour, between the electricity injected into the



Surplus of consumption



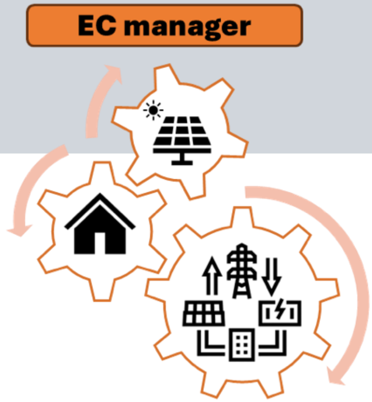
Surplus of production

Renewable Energy Communities

Operation Management

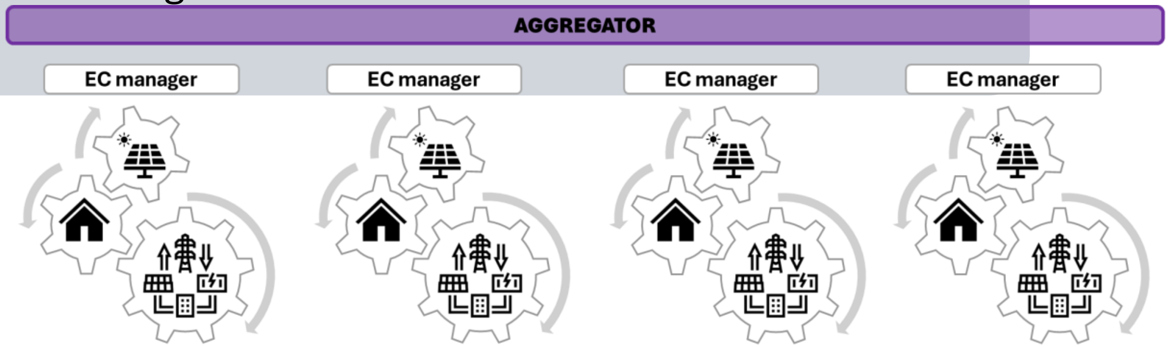
Internal Operations

- Shared energy maximization
- Optimization of each ECPs power exchange with the grid
- KKT conditions



External Operations

- Efficient participation of multiple RECs in the DR markets
- Optimization of each RECs power exchange with the grid
- KKT conditions



ECPs’ operational constraints

Power Balance

$$p_{i,t}^{G,in} - p_{i,t}^{G,out} + P_{i,t}^{PV} + p_{i,t}^{S,dch} - p_{i,t}^{S,ch} = P_{i,t}^{L,fix} + p_{i,t}^{L,flex} + p_{i,t}^{EV} \qquad i \in S^N, t \in S^T$$

Electric Vehicles

$$x_{i,t}^{EV} = x_{i,t-1}^{EV} + \frac{\Delta}{CAP_i^{EV}} \Gamma_i^{EV} p_{i,t}^{EV} \qquad i \in S^N, t \in S^T$$

$$0 \le p_{i,t}^{EV} \le \overline{P}_i^{EV} \qquad i \in S^N, t \in S^T$$

$$\underline{X}_i^{EV} \le x_{i,t}^{EV} \le \overline{X}_i^{EV} \qquad i \in S^N, t \in S^T$$

$$x_{i,t}^{EV} \ge X_i^{EV*} \qquad i \in S^N, t = T_i^{EV*}$$

ESS

$$x_{i,t}^{S} = x_{i,t-1}^{S} + \frac{\Delta}{CAP_i^{S}} \left(\Gamma_i^{S,ch} p_{i,t}^{S,ch} - \frac{1}{\Gamma_i^{S,dch}} p_{i,t}^{S,dch} \right) \qquad i \in S^N, t \in S^T$$

$$0 \le p_{i,t}^{S,ch} \le \overline{P}_i^{S} \qquad i \in S^N, t \in S^T$$

$$0 \le p_{i,t}^{S,dch} \le \overline{P}_i^{S} \qquad i \in S^N, t \in S^T$$

$$\underline{X}_i^{S} \le x_{i,t}^{S} \le \overline{X}_i^{S} \qquad i \in S^N, t \in S^T$$

Main Grid

$$0 \le p_{i,t}^{G,out} \le \overline{P}_i^{G} \qquad i \in S^N, t \in S^T$$

$$0 \le p_{i,t}^{G,in} \le \overline{P}_i^{G} \qquad i \in S^N, t \in S^T$$

Flexible Loads

$$\sum_{t \in T} p_{i,t}^{L,flex} \Delta \ge E_i^{L,flex} \qquad i \in S^N, t \in S^T$$

$$0 \le p_{i,t}^{L,flex} \le \overline{P}_i^{L,flex} \qquad i \in S^N, t \in S^T$$

RENEWABLE ENERGY COMMUNITIES

INTERNAL OPERATIONS

REC Bilevel Optimal Management

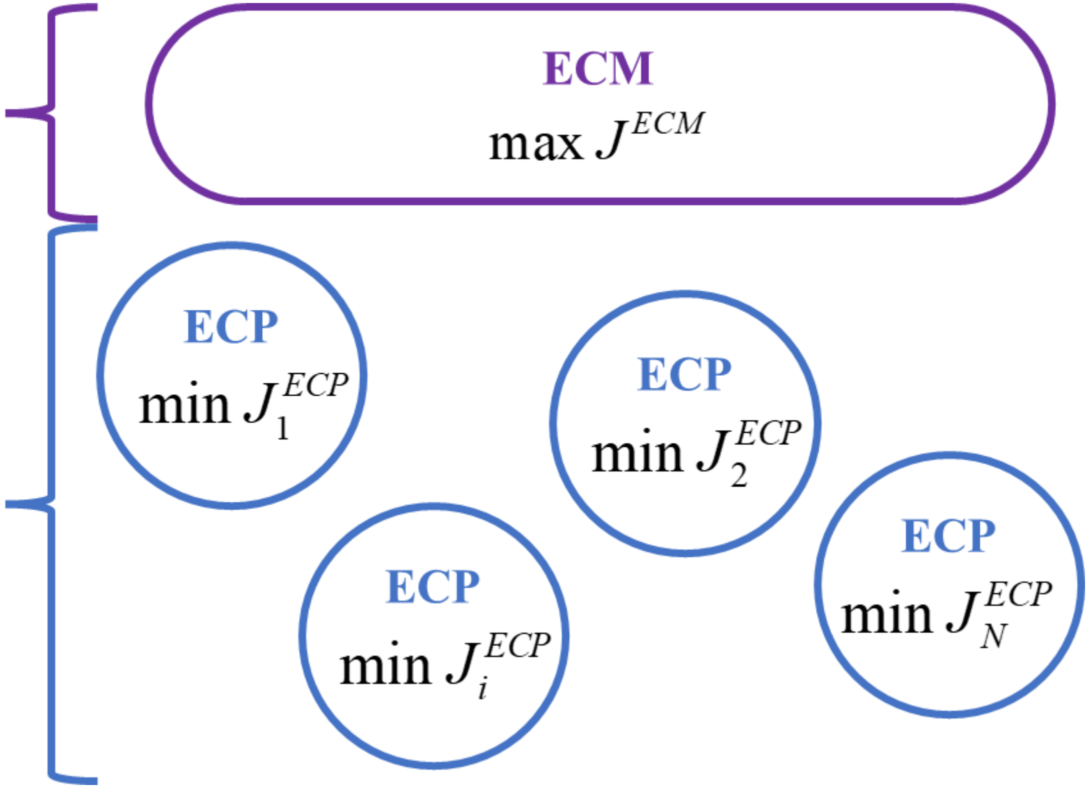
V. Casella, G. Ferro, L. Parodi, M. Robba. (2025) "Maximizing shared benefits in renewable energy communities: A Bilevel optimization model", *Applied Energy*, 386, 125562.

- Shared energy maximization
  - according to the **newest incentive scheme**

- Cost minimization

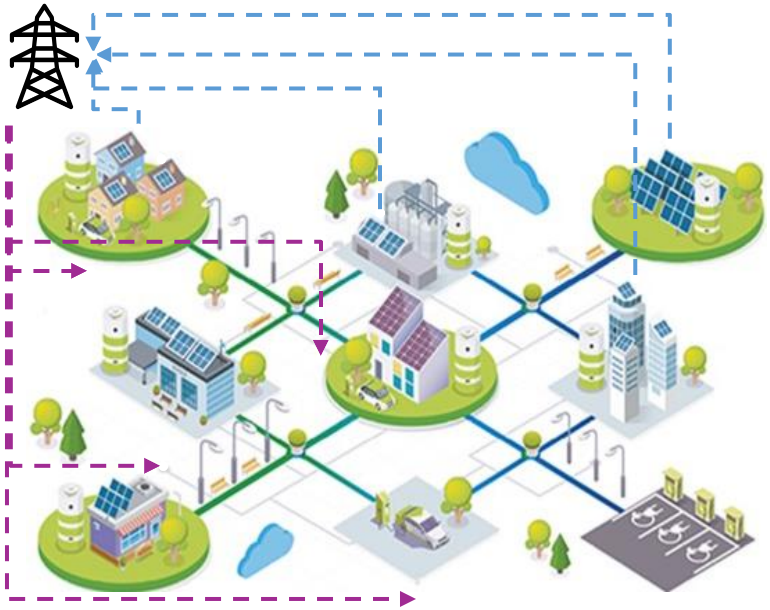
$$J_i^{ECP} = \Delta \sum_{t \in S^T} \left(C_t^{buy} p_{i,t}^{G,in} - C_t^{sell} p_{i,t}^{G,out} \right) + \alpha \left(x_T^S - x_0^S \right)^2$$

- ECPs' operational constraints



REC Bilevel Optimal Management

New incentive scheme



The incentive on shared energy it is made up of a fixed part and a variable part:

- the **fixed part** varies according to the size of the plant
- the **variable part** depends on the market price of energy

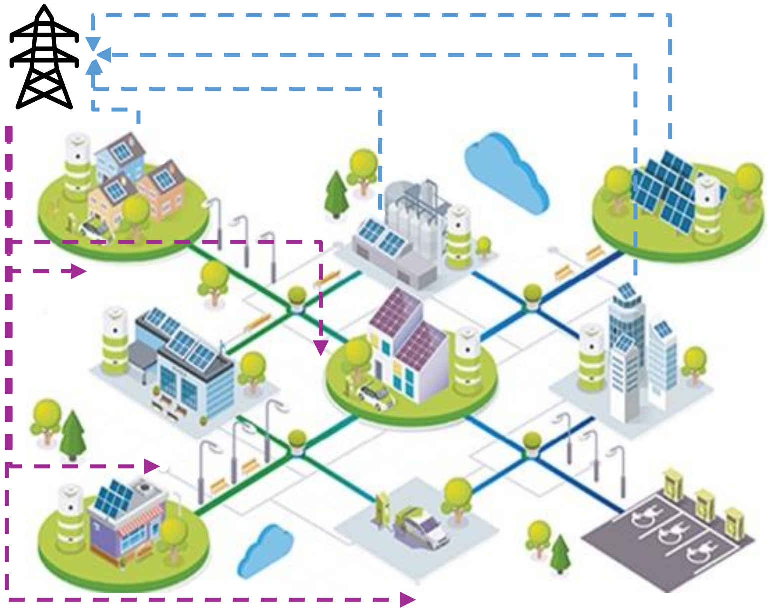
$$TIP_{i,t} = min\{CAP_i; TP_i^{base} + max\{0; 180 - Pz_t\} + FC_{zone}\} \cdot (1 - F)$$

[1] Decreto CER.pdf (mase.gov.it)

[2] DECRETO CACER e TIAD – Regole operative per l'accesso al servizio per l'autoconsumo diffuso e al contributo PNRR (mase.gov.it)

REC Bilevel Optimal Management

New incentive scheme



$$TIP_{i,t} = min\{CAP_i; TP_i^{base} + max\{0; 180 - Pz_t\} + FC_{zone}\} \cdot (1 - F)$$

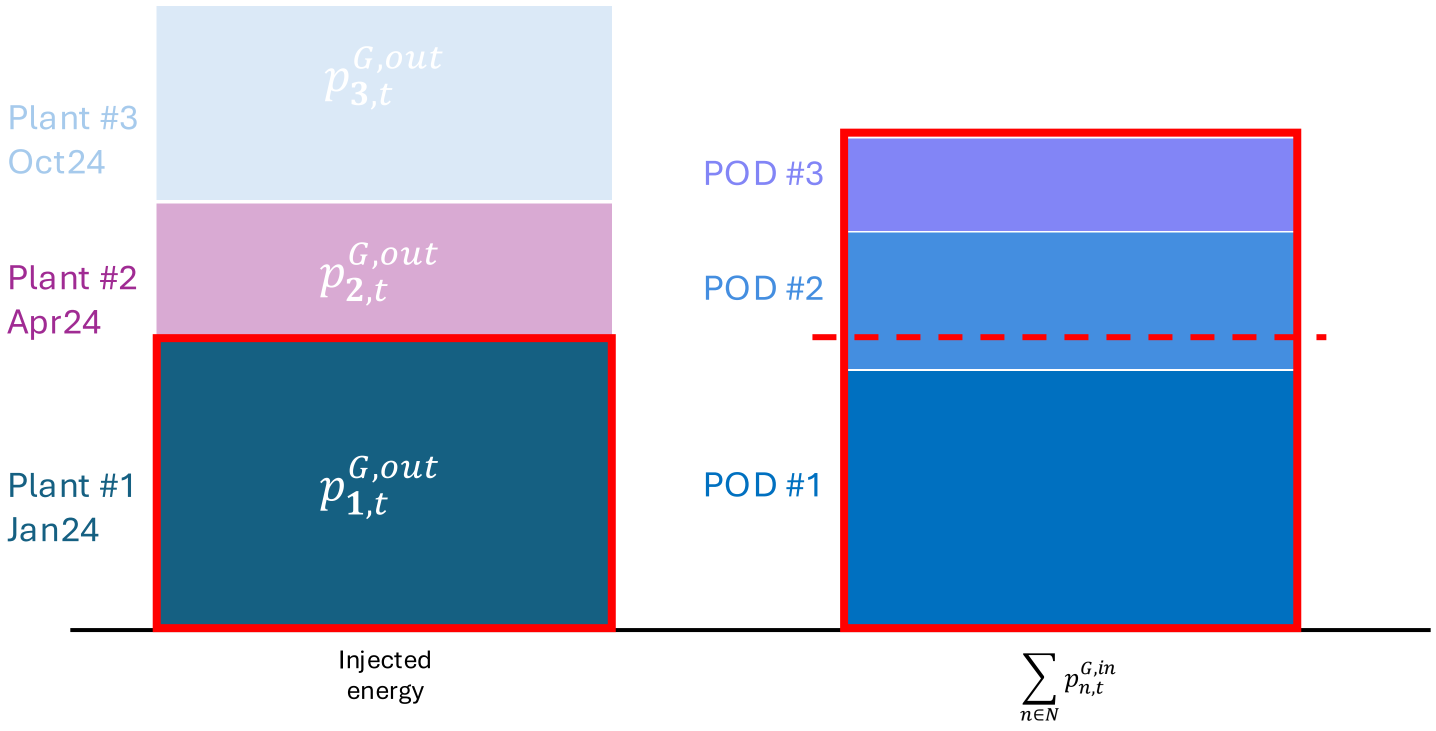
Each production plant has an incentive depending on its **size**.

However, since the shared energy is determined by different contributions from **different production plants**, the overall incentive depends on which plants are considered in the definition of the shared energy. Indeed, some plants or a portion of them can be excluded.

The rule coming from the legislation is that plants are ordered according to the **date of the first connection to the grid**.

REC Bilevel Optimal Management

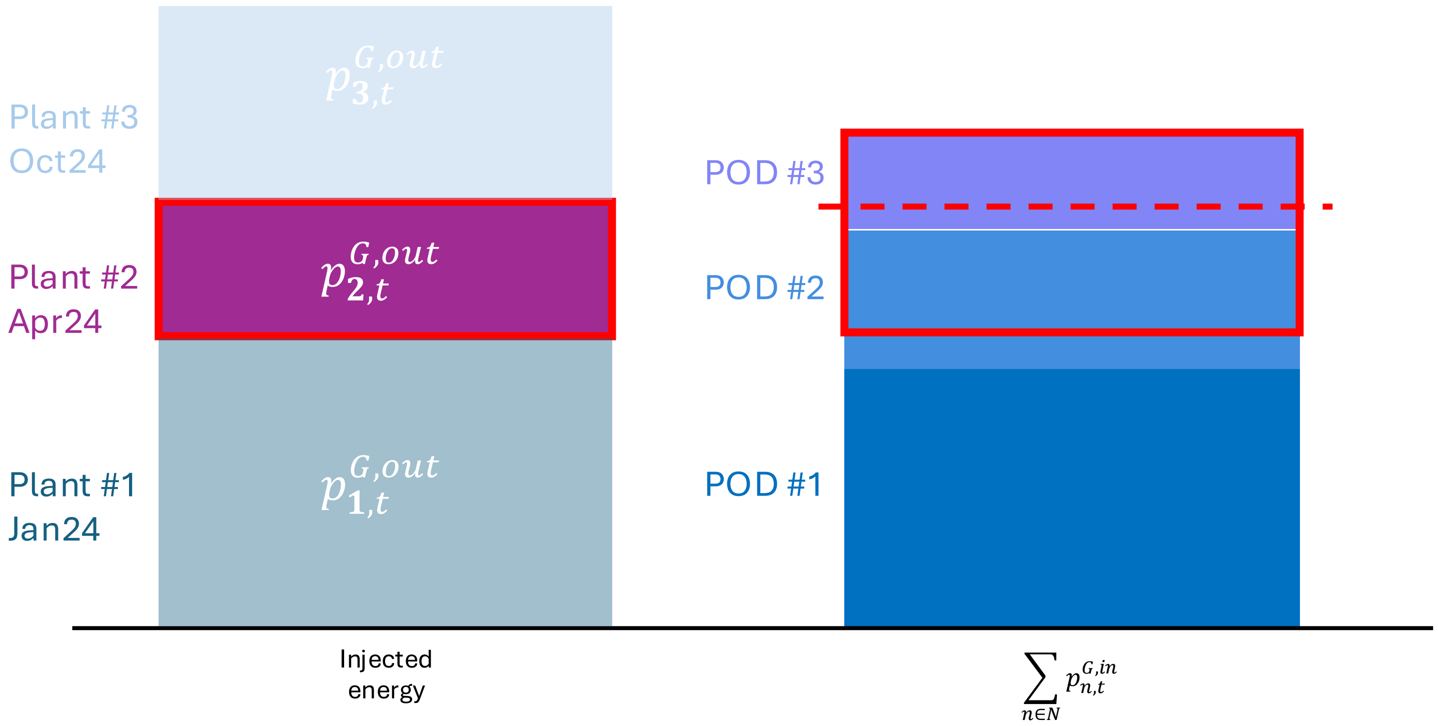
$$TIP_{i,t} = min\{CAP_i; TP_i^{base} + max\{0; 180 - Pz_t\} + FC_{zonale}\} \cdot (1 - F)$$



Plant #1 $TIP_{1,t} \Delta \cdot \min\left\{p_{1,t}^{g,out}, \max\left[0, \sum_{n \in N} p_{n,t}^{g,in}\right]\right\}$ → $p_{1,t}^{G,out}$

REC Bilevel Optimal Management

$$TIP_{i,t} = min\{CAP_i; TP_i^{base} + max\{0; 180 - Pz_t\} + FC_{zonale}\} \cdot (1 - F)$$



Plant #2 $TIP_{2,t} \Delta \cdot \min\left\{ p_{2,t}^{g,out}, \max\left[0, \sum_{n \in N} p_{n,t}^{g,in} - p_{1,t}^{g,out} \right] \right\} \rightarrow p_{2,t}^{G,out}$

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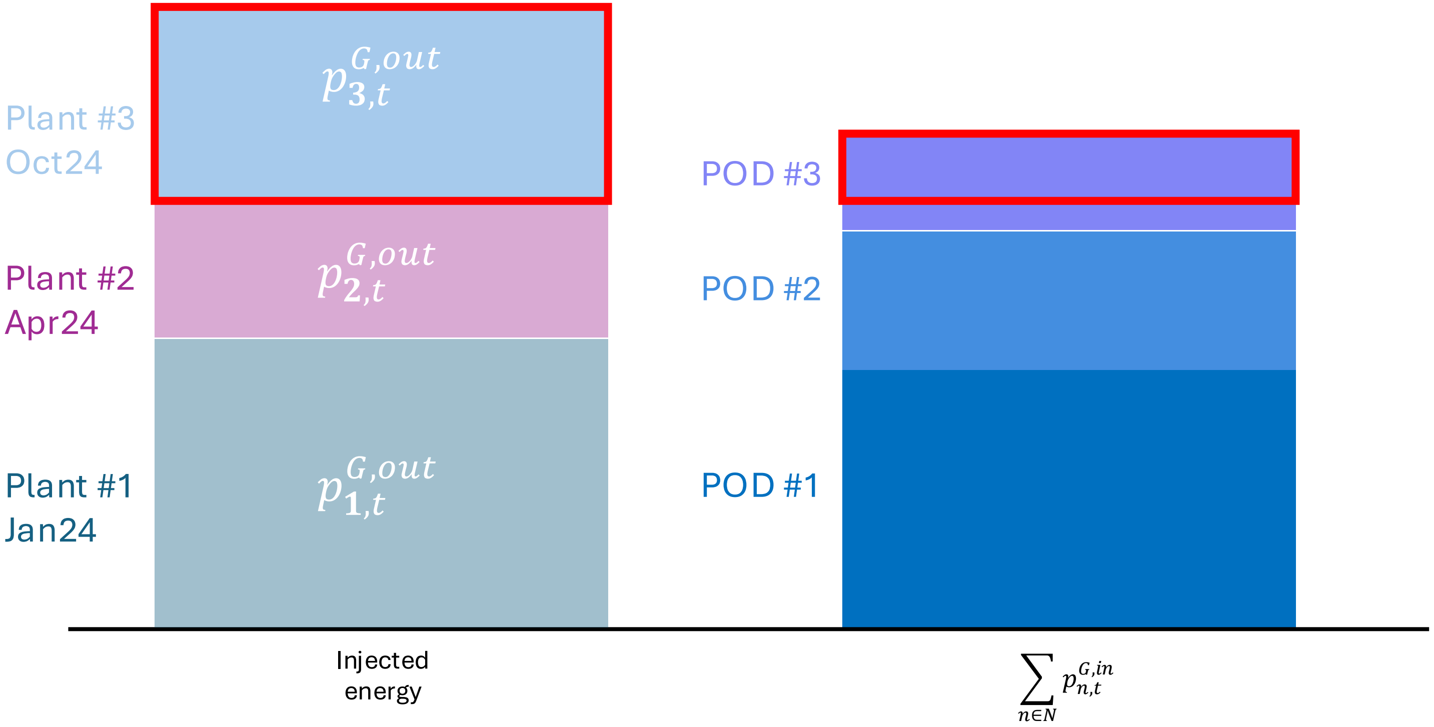
REC Bilevel Optimal Management

$$\boldsymbol{TIP_{i,t} = min\{CAP_i; TP_i^{base} + max\{0; 180 - Pz_t\} + FC_{zonale}\} \cdot (1 - F)}$$



Plant #3 $TIP_{3,t}\Delta \cdot \min\left\{p_{3,t}^{g,out}, \max\left[0, \sum_{n \in N} p_{n,t}^{g,in} - \left(p_{1,t}^{g,out} - p_{2,t}^{g,out}\right)\right]\right\} \longrightarrow \sum_{n \in N} p_{n,t}^{G,in} - \left(p_{1,t}^{G,out} - p_{2,t}^{G,out}\right)$

REC Bilevel Optimal Management

Plant #1 $TIP_{1,t}\Delta \cdot \min\left\{p_{1,t}^{g,out}, \max\left[0, \sum_{n\in N} p_{n,t}^{g,in}\right]\right\}$

Plant #2 $TIP_{2,t}\Delta \cdot \min\left\{p_{2,t}^{g,out}, \max\left[0, \sum_{n\in N} p_{n,t}^{g,in} - p_{1,t}^{g,out}\right]\right\}$

Plant #3 $TIP_{3,t}\Delta \cdot \min\left\{p_{3,t}^{g,out}, \max\left[0, \sum_{n\in N} p_{n,t}^{g,in} - \left(p_{1,t}^{g,out} - p_{2,t}^{g,out}\right)\right]\right\}$

⋮

REC incentive

$$\sum_{p\in P} TIP_{p,t}\Delta \cdot \min\left\{p_{p,t}^{g,out}, \max\left[0, \sum_{n\in N} p_{n,t}^{g,in} - \sum_{i=1}^{p-1} p_{i,t}^{g,out}\right]\right\}$$

ECM objective

$$\max \sum_{t=0}^{T-1}\left\{\sum_{p\in P} TIP_{p,t}\Delta \cdot \min\left\{p_{p,t}^{g,out}, \max\left[0, \sum_{n\in N} p_{n,t}^{g,in} - \sum_{i=1}^{p-1} p_{i,t}^{g,out}\right]\right\}\right\}$$

REC Bilevel Optimal Management

$$\max \sum_{t=0}^{T-1} \left\{ \sum_{p \in P} TIP_{p,t} \Delta \cdot \min \left\{ p_{p,t}^{g,out}, \max \left[0, \sum_{n \in N} p_{n,t}^{g,in} - \sum_{i=1}^{p-1} p_{i,t}^{g,out} \right] \right\} \right\}$$

REC Bilevel Optimal Management

$$\max \sum_{t=0}^{T-1} \left\{ \sum_{p \in P} TIP_{p,t} \Delta \cdot \min \left\{ p_{p,t}^{g,out}, \max \left[0, \sum_{n \in N} p_{n,t}^{g,in} - \sum_{i=1}^{p-1} p_{i,t}^{g,out} \right] \right\} \right\}$$

$$\begin{cases} \max \sum_{t=1}^{T} \left\{ \sum_{p \in P} TIP_{p,t} \Delta \cdot \min \left\{ p_{p,t}^{g,out}, \beta_{p,t} \right\} \right\} \\ \beta_{p,t} \geq \sum_{n \in N} p_{n,t}^{g,in} - \sum_{i=1}^{p-1} p_{i,t}^{g,out} \\ \beta_{p,t} \geq 0 \end{cases}$$

$$\min(x_1, x_2) = -\max(-x_1, -x_2) \longrightarrow \max \sum_{t=1}^{T} \left\{ \sum_{p \in P} TIP_{p,t} \Delta \cdot \left[-\max \left\{ -p_{p,t}^{g,out}, -\beta_{p,t} \right\} \right] \right\}$$

REC Bilevel Optimal Management

$$\max \sum_{t=1}^{T} \left\{ \sum_{p \in P} TIP_{p,t} \Delta \cdot \left[-\max\left\{ -p_{p,t}^{g,out}, -\beta_{p,t} \right\} \right] \right\}$$

a. MILP problem
b. NLP problem

REC Bilevel Optimal Management

$$\max \sum_{t=1}^{T} \left\{ \sum_{p \in P} TIP_{p,t} \Delta \cdot \left[-\max\left\{ -p_{p,t}^{g,out}, -\beta_{p,t} \right\} \right] \right\}$$

a. MILP problem
b. NLP problem

$$\max\left(x_1, x_2\right) = \frac{\left(x_1 + x_2\right) + \left|x_1 - x_2\right|}{2}$$

$$\max \sum_{t=1}^{T} \left\{ \sum_{p \in P} TIP_{p,t} \Delta \cdot \left[-\frac{\left(-p_{p,t}^{g,out} - \beta_{p,t}\right) + \left|-p_{p,t}^{g,out} - \beta_{p,t}\right|}{2} \right] \right\}$$

REC Bilevel Optimal Management

$$\max \sum_{t=1}^{T} \left\{ \sum_{p \in P} TIP_{p,t} \Delta \cdot \left[-\max\left\{ -p_{p,t}^{g,out}, -\beta_{p,t} \right\} \right] \right\}$$

a. MILP problem

b. NLP problem[3]

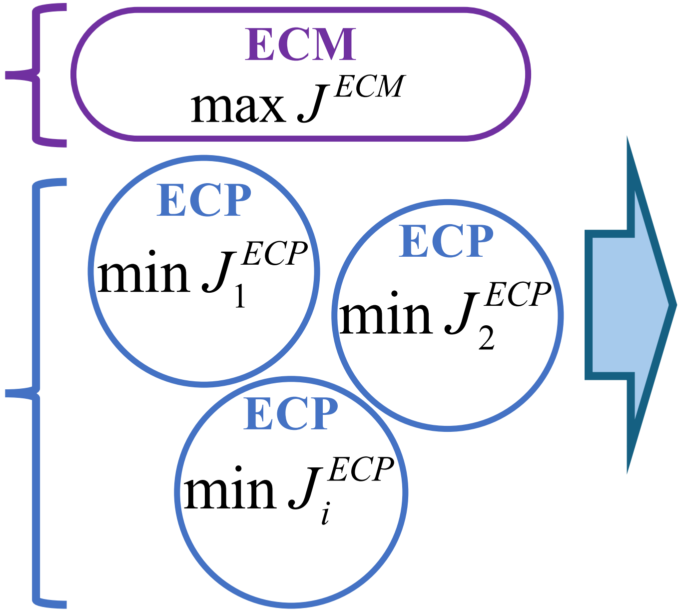
$$erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$$

$$\max(x_1, x_2) = f(x_1, x_2, \mu) = \frac{(x_1 + x_2) + (x_1 - x_2)\, erf\left(\mu(x_1 - x_2)\right)}{2}$$

$$\max \sum_{t=1}^{T} \left\{ \sum_{p \in P} TIP_{p,t} \Delta \cdot \left[-\frac{\left(-p_{p,t}^{g,out} - \beta_{p,t}\right) + \left(-p_{p,t}^{g,out} + \beta_{p,t}\right) erf\left(\mu\left(-p_{p,t}^{g,out} - \beta_{p,t}\right)\right)}{2} \right] \right\}$$

[3] K. Biswas, S. Kumar, S. Banerjee, e A. K. Pandey, «Smooth Maximum Unit: Smooth Activation Function for Deep Networks using Smoothing Maximum Technique», in 2022 IEEE/CVF Conference on Computer Vision and Pattern Recognition (CVPR), New Orleans, LA, USA: IEEE, giu. 2022, pp. 784–793.

REC Bilevel Optimal Management



$$\max J^{ECM}$$

$s.t.$

KKT conditions for ECP_1

KKT conditions for ECP_2

M

KKT conditions for ECP_i

M

MILP

LP Formulation + Linearized KKT constraints for each ECPs

NLP

NLP Formulation + KKT constraints for each ECPs

REC Bilevel Optimal Management

We have done some tests using the YALMIP toolbox in MATLAB.

The two solvers are GUROBI (MILP) and IPOPT (NLP).

	T=24 N=3		T=24 N=6		T=24 N=10		T=24 N=15		T=24 N=20		T=24 N=30		T=24 N=50	
	MILP	**NLP**	**MILP**	**NLP**	**MILP**	**NLP**	**MILP**	**NLP**	**MILP**	**NLP**	**MILP**	**NLP**	**MILP**	**NLP**
Solver time [s]	21,17	4,58	285,31	12,32	>1000	27,99	------	79,81	------	79,75	------	175,52	------	699,7
Obj	28,27	28,27	28,64	28,64		28,64	------	31,03	------	31,03	------	59,04	------	89,44
Err %		-5.69E-05		-1.08E-04										

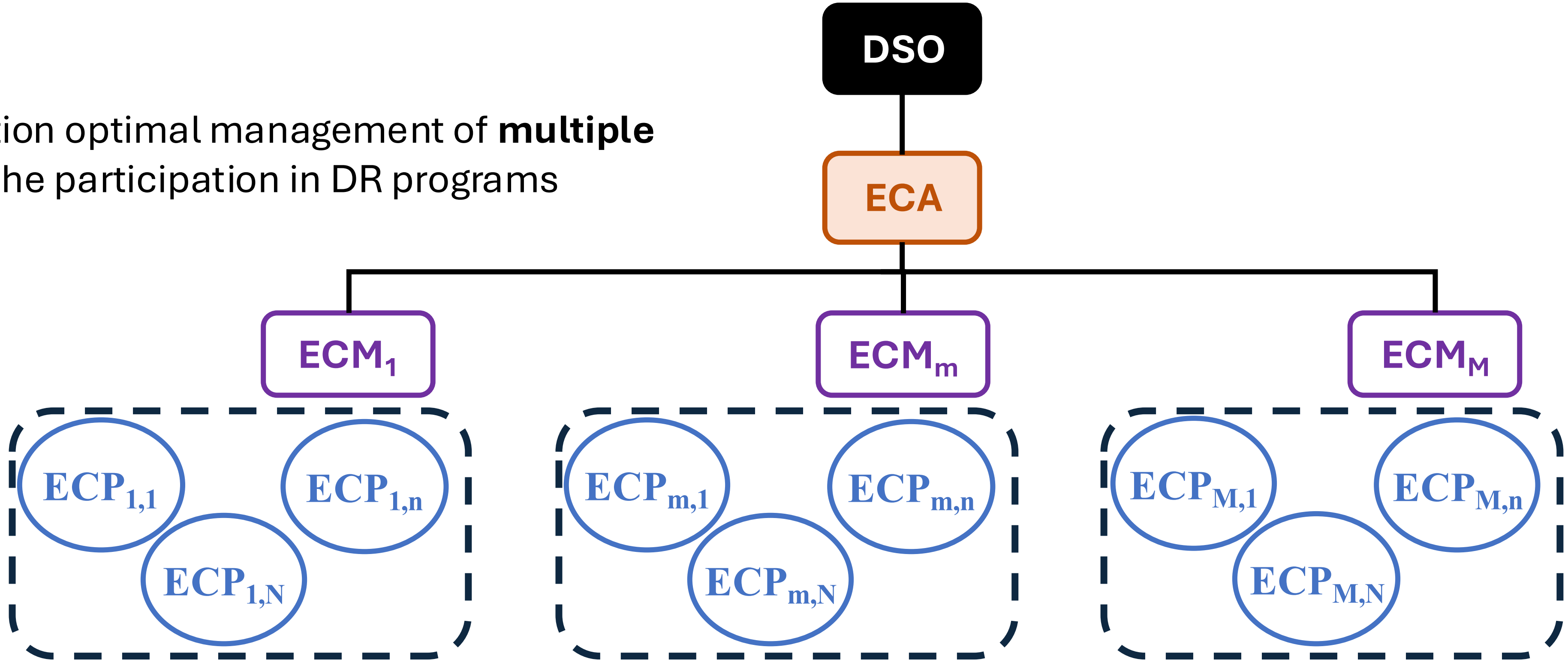
RENEWABLE ENERGY COMMUNITIES

EXTERNAL OPERATIONS

Multiple REC optimization for DR services

V. Casella, L. Farina, G. Ferro, L. Parodi, M. Robba "Operational management of multiple energy communities in the energy market: a bilevel optimization-based approach", IFAC2025

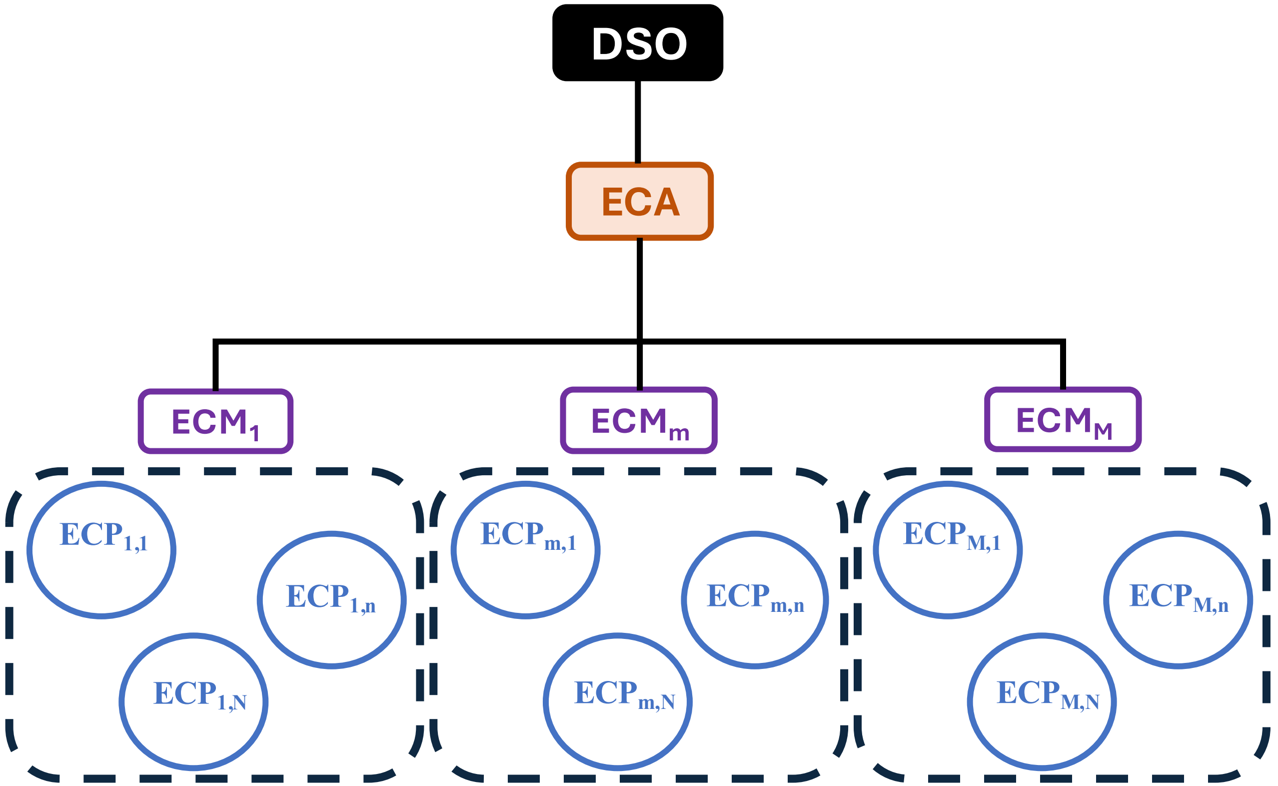
Coordination optimal management of **multiple RECs** for the participation in DR programs



Multiple REC optimization for DR services

- Tracking problem of the DR reference power value from the DSO

- Shared energy maximization + ECP cost minimization
- ECPs' operational constraints



Multiple REC optimization for DR services

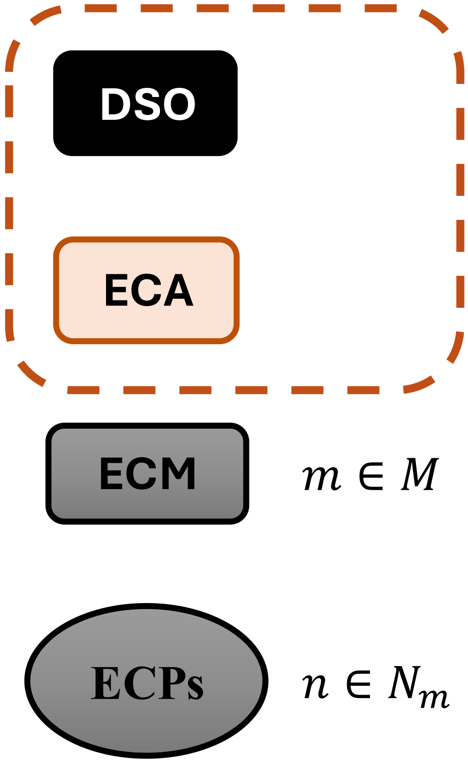
The high level models the ECA **tracking** the reference DR power value from the DSO

DR targeting power

$$J^{eca} = \sum_{t \in T^{dr}} \left(P_t^{dr} - \sum_{m \in M} p_{m,t}^{ec,g} \right)^2$$

REC power exchange

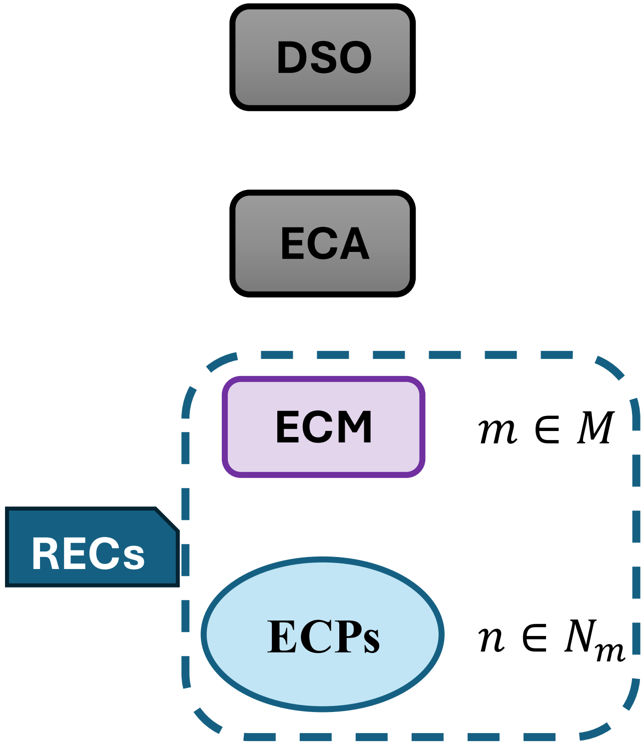
DR reference power value from DSO



Multiple REC optimization for DR services

The low level models multiple RECs, each one characterized by ECPs and an ECM.

$$J_m^{ec} = J_m^{ecm} + \sum_{n \in N_m} J_{m,n}^{ecp} \quad m \in M$$



Multiple REC optimization for DR services

The low level models multiple RECs, each one characterized by ECPs and an ECM.

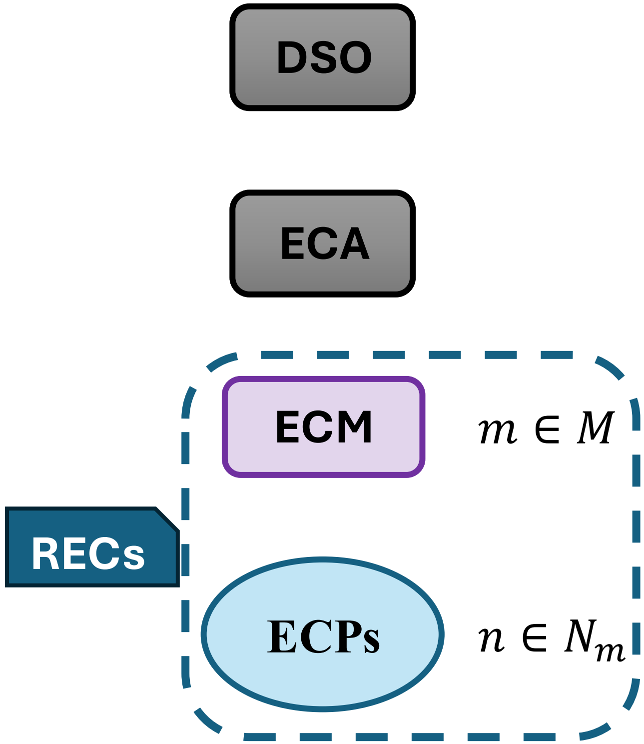
$$J_m^{ec} = J_m^{ecm} + \sum_{n \in N_m} J_{m,n}^{ecp} \quad m \in M$$

Shared energy maximization

$$J_m^{ecm} = -\Delta C^{inc} \sum_{t \in T} \beta_{m,t} \quad m \in M$$

$$\beta_{m,t} \leq \sum_{n \in N_m} p_{m,n,t}^{out} \quad m \in M, t \in T$$

$$0 \leq \beta_{m,t} \leq \sum_{n \in N_m} p_{m,n,t}^{in} \quad m \in M, t \in T$$



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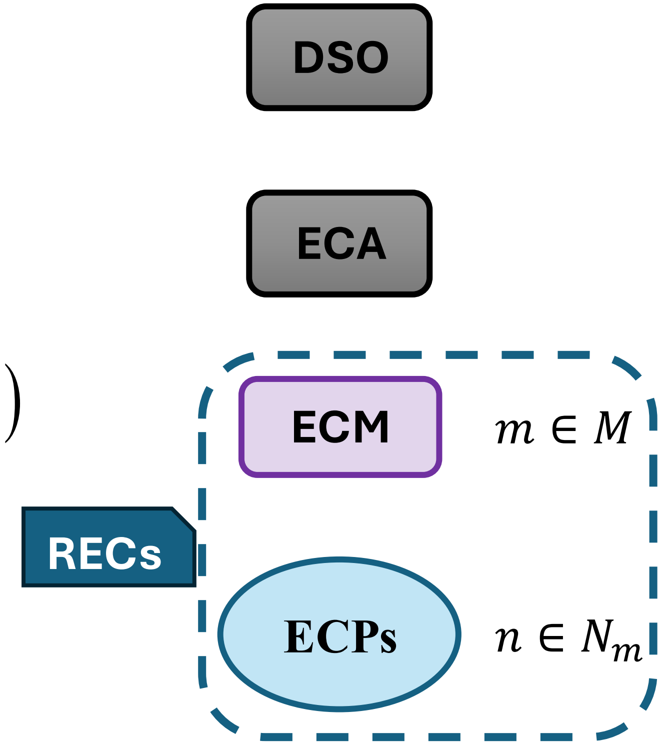
$$J_m^{ec} = J_m^{ecm} + \sum_{n \in N_m} J_{m,n}^{ecp} \qquad m \in M$$

ECPs' cost minimization

$$J_{m,n}^{ecp} = \Delta \sum_{t \in T} \left(C_{m,n,t}^{buy} p_{m,n,t}^{in} - C_{m,n,t}^{sell} p_{m,n,t}^{out} \right)$$

ECPs operational constraints



Multiple REC optimization for DR services

The low level models multiple RECs, each one characterized by ECPs and an ECM.

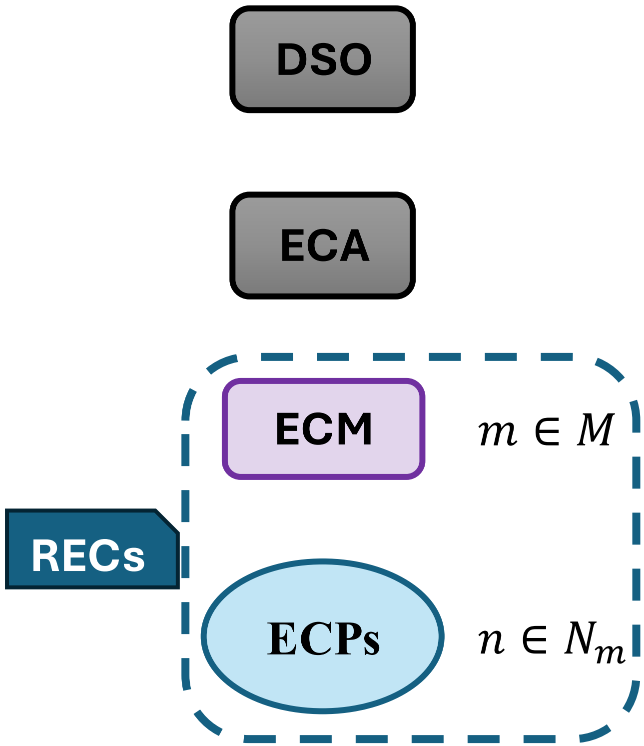
$$\boldsymbol{\Omega}\mathbf{u} + \boldsymbol{\Psi} \leq \mathbf{0}$$

$$\boldsymbol{\Omega} = \begin{bmatrix} \boldsymbol{\Omega}_1 & \mathbf{0} & & \Box & & \mathbf{0} \\ \mathbf{0} & \Box & & & & \\ & & \boldsymbol{\Omega}_n & & & \Box \\ \Box & & & \Box & & \\ \mathbf{0} & & \Box & \mathbf{0} & \boldsymbol{\Omega}_N & \mathbf{0} \\ \boldsymbol{\Omega}_1^{\beta} & \Box & \boldsymbol{\Omega}_n^{\beta} & \Box & \boldsymbol{\Omega}_N^{\beta} & \boldsymbol{\Omega}^{\beta} \end{bmatrix} \quad \boldsymbol{\Psi} = \begin{bmatrix} \boldsymbol{\Psi}_1 \\ \Box \\ \boldsymbol{\Psi}_n \\ \Box \\ \boldsymbol{\Psi}_N \\ \boldsymbol{\Psi}^{\beta} \end{bmatrix}$$

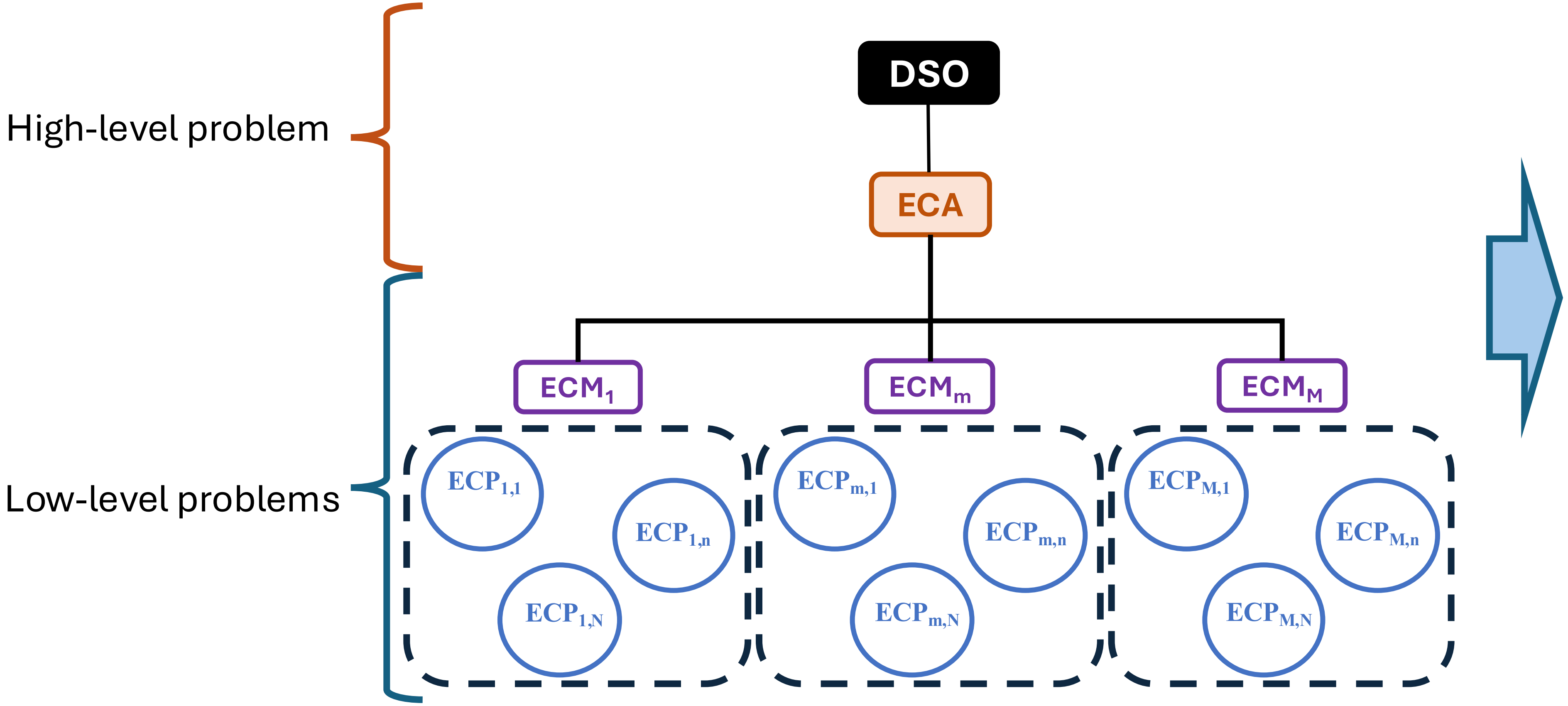
ECPs ($\boldsymbol{\Psi}_1 \ldots \boldsymbol{\Psi}_N$)

ECM ($\boldsymbol{\Psi}^{\beta}$)

$$\boldsymbol{\Omega}_n^{\beta} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{-I} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{-I} & \mathbf{-I} & \mathbf{-I} & \mathbf{I} & \mathbf{-I} \end{bmatrix} \quad n \in N, \quad \boldsymbol{\Omega}^{\beta} = \begin{bmatrix} \mathbf{-I} \\ \mathbf{I} \\ \mathbf{I} \end{bmatrix}, \quad \boldsymbol{\Psi}^{\beta} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ (\mathbf{P}^{pv} - \mathbf{P}^{fix})\mathbf{1}^{N\times 1} \end{bmatrix}$$



Multiple REC optimization for DR services



$$\max J^{ECA}$$

$s.t.$

KKT conditions for REC_1

KKT conditions for REC_2

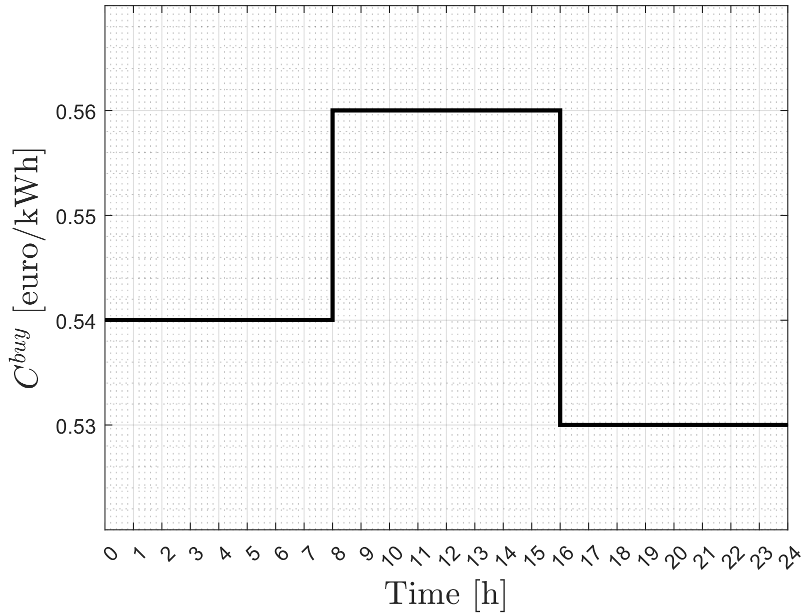
M

KKT conditions for REC_i

M

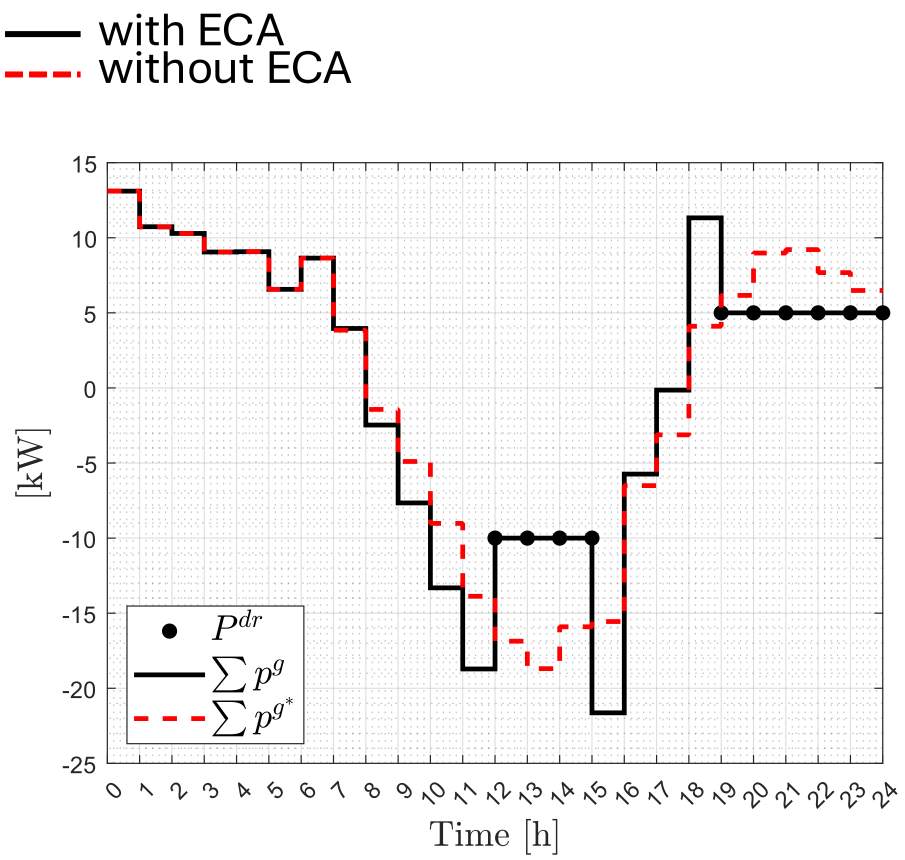
Multiple REC optimization for DR services

# ECs	# ECPs	Δ $[h]$	C^{inc} [€/kWh]	C^{sell} [€/kWh]
3	5,3,8	1	0.110	0.08



- 3-slots energy tariff
- NLP solved with MATLAB IPOPT in **130** seconds

Multiple REC optimization for DR services



ECA [kW^2]	ECMs [€]	ECPs [€]
0.00	-24.11	154.8

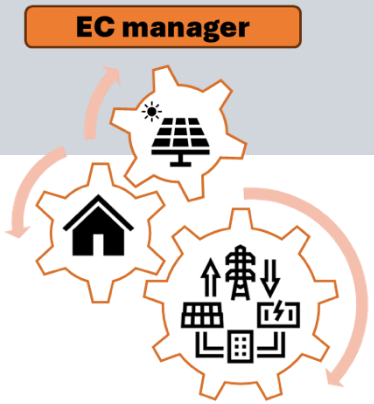
- Perfect matching with DR bid power with ECA
- Same EC costs with and without ECA, thanks to KKT conditions

Renewable Energy Communities

Operation Management

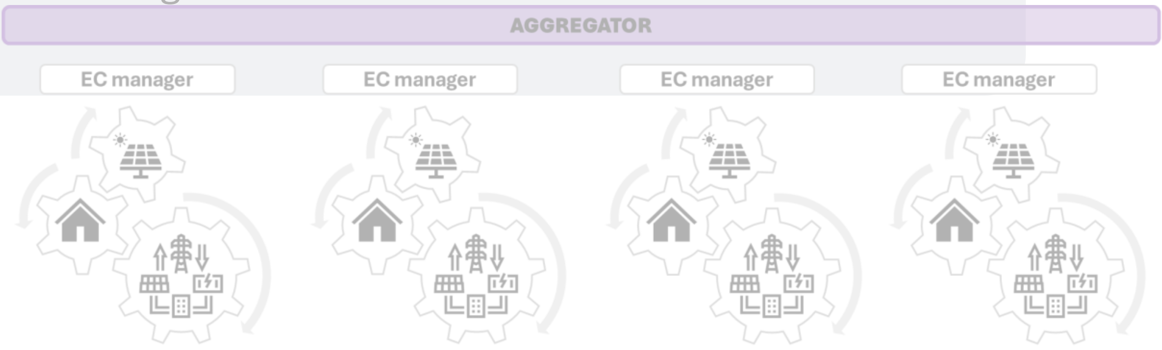
Internal Operations

- Shared energy maximization
- Optimization of each ECPs power exchange with the grid
- ~~KKT conditions~~



External Operations

- Efficient participation of multiple RECs in the DR markets
- Optimization of each RECs power exchange with the grid
- KKT conditions



REC Optimal Management

V. Casella, G. Ferro, L. Glielmo, L. Parodi, M. Robba. (-------). *Renewable Energy Communities Cooperation for Demand Response Services*. *Submitted to IEEE Transactions on Automation Science and Engineering*

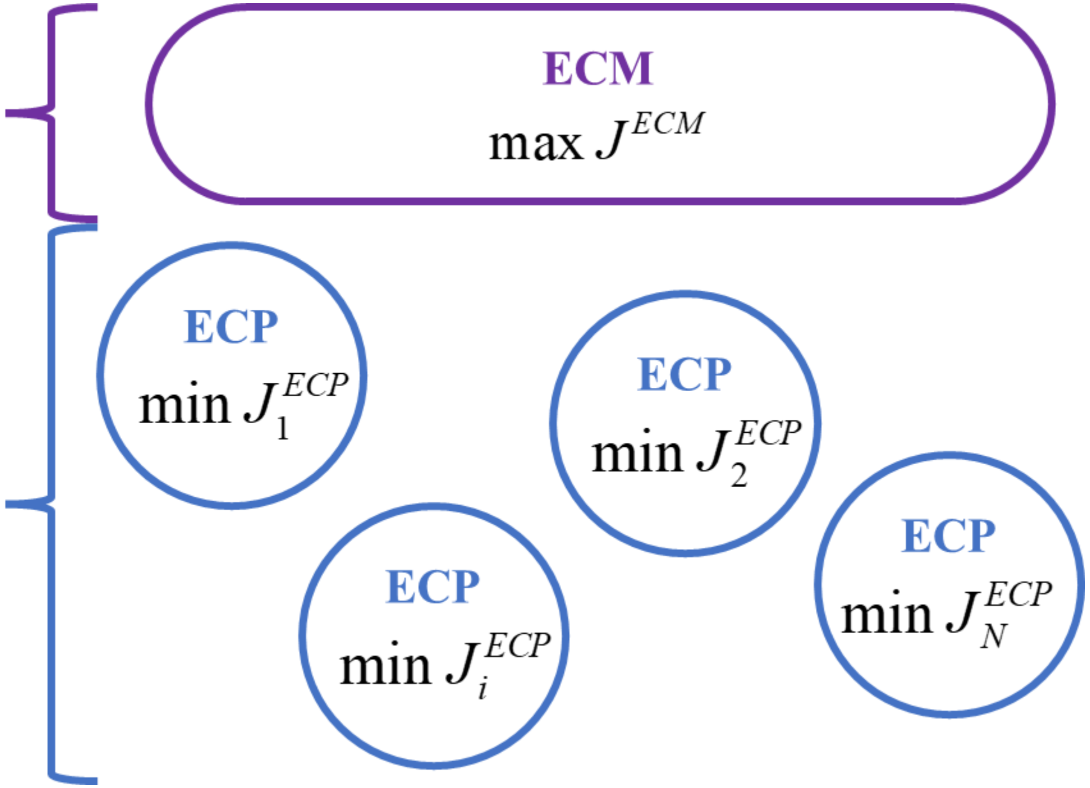
- Shared energy maximization

$$\max J^{ECM} = \sum_{t \in S^T} \min\left\{ \sum_{i \in S^M} p_{i,t}^{G,in}, \sum_{i \in S^M} p_{i,t}^{G,out} \right\} \Delta$$

- Cost minimization

$$\min J_i^{ECP} = \Delta \sum_{t \in S^T} \left(C_t^{buy} p_{i,t}^{G,in} - C_t^{sell} p_{i,t}^{G,out} \right)$$

- ECPs' operational constraints



REC Optimal Management

V. Casella, G. Ferro, L. Glielmo, L. Parodi, M. Robba. (-------). *Renewable Energy Communities Cooperation for Demand Response Services*. *Submitted to IEEE Transactions on Automation Science and Engineering*

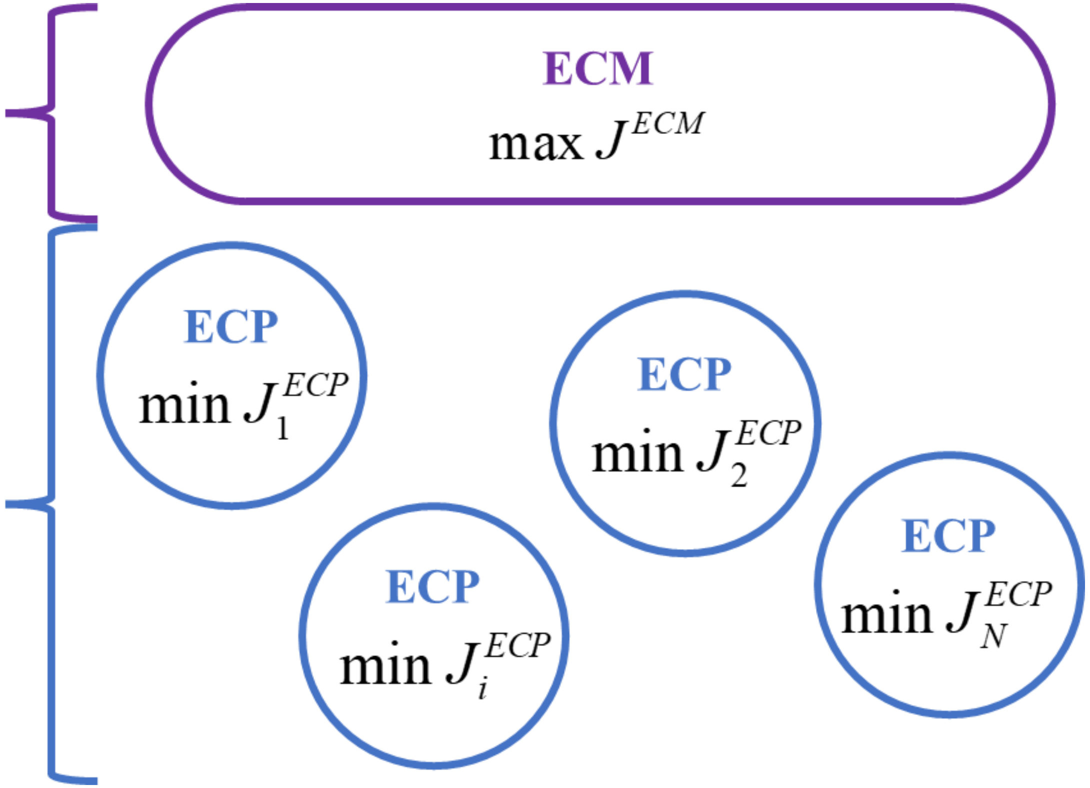
Shared energy maximization (Alternative formulation)

$$\min J^{ECM} = \sum_{t \in S^T} \left[\alpha_1 \left(\sum_{i \in S^M} \frac{p_{i,t}^{G,\text{in}} - p_{i,t}^{G,\text{out}}}{\overline{P}_i^G} \right)^2 - \alpha_2 \sum_{i \in S^M} \left(\frac{p_{i,t}^{G,\text{out}}}{\overline{P}_i^G} \right) \right]$$

Cost minimization

$$\min J_i^{ECP} = \Delta \sum_{t \in S^T} \left(C_t^{buy} p_{i,t}^{G,in} - C_t^{sell} p_{i,t}^{G,out} \right)$$

ECPs' operational constraints



Optimization Model

$$\min J = \sum_{t \in S^T} \left[\alpha_1 \left(\sum_{i \in S^M} \frac{p_{i,t}^{G,\text{in}} - p_{i,t}^{G,\text{out}}}{\overline{P}_i^G} \right)^2 - \alpha_2 \sum_{i \in S^M} \left(\frac{p_{i,t}^{G,\text{out}}}{\overline{P}_i^G} \right) + \alpha_3 \sum_{t \in S^M} \left(C_t^{buy} p_{i,t}^{G,in} - C_t^{sell} p_{i,t}^{G,out} \right) \Delta \right]$$

Constraints on Power Balance, EVs and Storage dynamics, flexible demands for each **ECP**

Constraints on Demand Response Policy

$$\sum_{i \in S^M} \left(p_{i,t}^{G,\text{in}} - p_{i,t}^{G,\text{out}} \right) = P_t^{DR} \qquad \forall t \in S^{DR} \subseteq S^T$$

Optimization Model

$$\min J = \sum_{t\in S^T}\left[\alpha_1\left(\sum_{i\in S^M}\frac{p_{i,t}^{G,\text{in}} - p_{i,t}^{G,\text{out}}}{\bar{P}_i^G}\right)^2 - \alpha_2\sum_{i\in S^M}\left(\frac{p_{i,t}^{G,\text{out}}}{\bar{P}_i^G}\right) + \alpha_3\sum_{t\in S^M}\left(C_t^{buy}p_{i,t}^{G,in} - C_t^{sell}p_{i,t}^{G,out}\right)\Delta\right]$$

$$u = \text{col}(u_i)$$

$$Q = \text{diag}(Q_i)$$

$$q = \text{row}(q_i)$$

$$u_i = \begin{bmatrix} p^{G,\text{out}} & p^{S,\text{ch}} & p^{S,\text{dch}} & p^{L,\text{flex}} & p^{\text{EV}} \end{bmatrix}_i$$

$$Q_i = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & I & -I & I & I \\ 0 & -I & I & -I & -I \\ 0 & I & -I & I & I \\ 0 & I & -I & I & I \end{bmatrix}$$

$$q_i^T = \frac{1}{\bar{P}_i^G}\left(P^{L,fix} - P^{PV}\right)^T\begin{bmatrix}0 & I & -I & I & I\end{bmatrix} + \\ -\begin{bmatrix}\frac{1}{\bar{P}_i^G} & 0 & 0 & 0 & 0\end{bmatrix} + \\ +\begin{bmatrix}(C_i^{\text{buy}} - C_i^{\text{sell}}) & C_i^{\text{buy}} & -C_i^{\text{buy}} & C_i^{\text{buy}} & C_i^{\text{buy}}\end{bmatrix}$$

Optimization Model

$$\min_{u} \quad \frac{1}{2}u^T Q u + q^T u$$

$$s.t.$$

$$Cu - d \leq 0$$

$$Au - b = 0$$ ⬅ Demand Response

Optimization Algorithm

$$\min_{u} \quad f(u) \quad \Rightarrow \quad f(u): ¡^{N} \rightarrow ¡$$ <u>convex</u>

$s.t.$

$$Cu - d \leq 0$$

$$Au - b = 0$$

Define the Lagrangian

$$L(u, v, \mu) = f(u) + v^{T}(Au - b) + \mu^{T}(Cu - d)$$

v Equalities multiplier

μ Inequalities multiplier

Optimization Algorithm

$$\min_u \quad f(u) \quad \Rightarrow \quad f(u): \mathbb{R}^N \to \mathbb{R}$$ <u>convex</u>

$$s.t.$$

$$Cu - d \leq 0$$

$$Au - b = 0$$

Define the Lagrangian

$$\mathcal{L}(u,v,\mu) = L(u,v,\mu) + \frac{1}{2}(u^T C^T - d^T) I^\rho (Cu - d)$$

v Equalities multiplier

μ Inequalities multiplier

Bertsekas, D. P. (2014). *Constrained optimization and Lagrange multiplier methods*. Academic press.

$$\mathcal{L}(u, \mu_k) =$$

$$f(u) + \frac{1}{2\rho} \sum_{i \in R(C)} \left\{ \left[\max\left\{0, \mu_{ik} + \rho\left(C_i u - d_i\right)\right\} \right]^2 - \mu_{ik}{}^2 \right\}$$

$$\Downarrow$$

$$I^\rho_{i,i} = \rho > 0 \quad \text{if} \quad \mu_i > 0, \quad i \in R(C)$$

Optimization Algorithm

$$\min_u \quad f(u) \quad \Rightarrow \quad f(u): \mathbb{R}^N \to \mathbb{R} \quad \underline{\text{convex}}$$

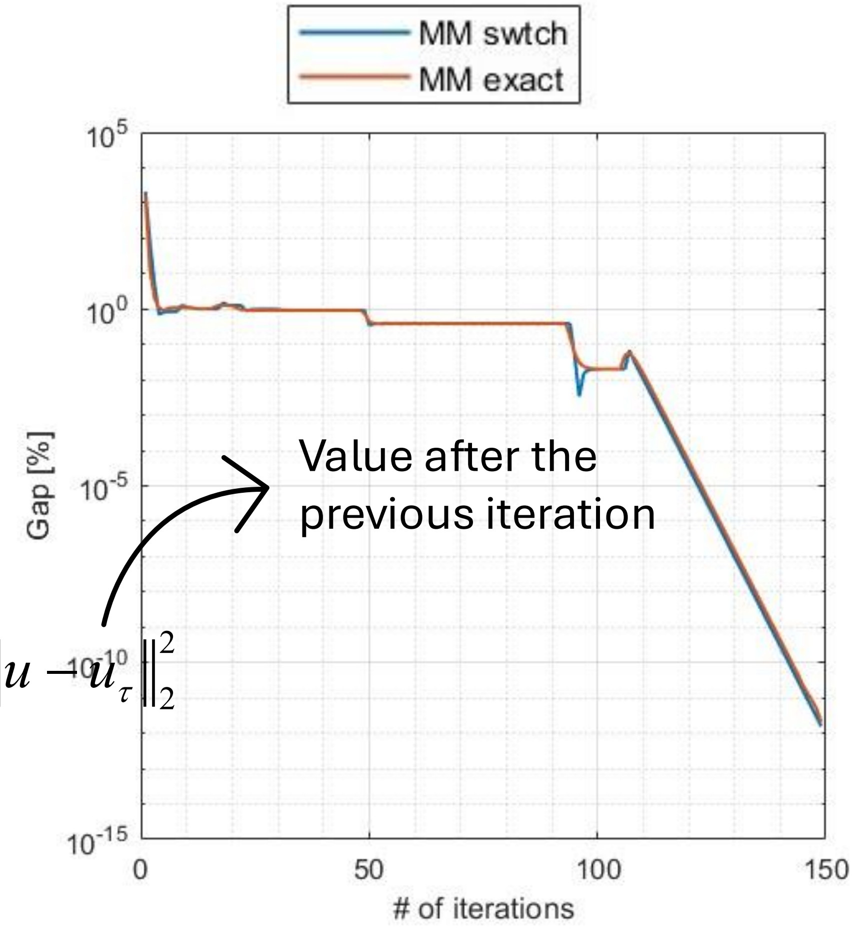
$s.t.$

$$Cu - d \le 0$$

$$Au - b = 0$$

Define the Lagrangian

$$\mathcal{L}(u,\nu,\mu) = L(u,\nu,\mu) + \frac{1}{2}(u^T C^T - d^T) I^\rho (Cu - d) + \frac{1}{2\rho}\|u - u_\tau\|_2^2$$



ν Equalities multiplier

μ Inequalities multiplier

Optimization Algorithm

First order conditions

$$\nabla_u \mathrm{L}(u,\nu,\mu) = \nabla_u f(u) + A^T \nu + C^T \mu + C^T I^\rho C u - C^T I^\rho d + \frac{1}{\rho}(u - u_\tau) = 0.$$

In view of the implicit function theorem, the solution of the first order conditions system $u(\nu,\mu)$ is a continuous and differentiable function if $\nabla^2_{uu}\mathrm{L}(u,\nu,\mu)$ is invertible (in a neighborhood of u^*)

$$\nabla^2_{uu}\mathrm{L}(u,\nu,\mu) = \underbrace{\nabla_{uu} f(u) + C^T I^\rho C + \frac{1}{\rho} I}_{\text{positive definite}}$$

Optimization Algorithm

Derive the second order dual update. We start defining the dual function

$$\mathrm{D}(\nu,\mu_\tau) = \mathrm{L}\left(u\left(\nu,\mu_\tau\right),\nu,\mu_\tau\right)$$

Then, we compute the gradient and the Hessian

$$\nabla_\nu \mathrm{D}(\nu,\mu_\tau) = Au(\nu,\mu_\tau) - b$$

$$\nabla^2_{\nu\nu}\mathrm{D}(\nu,\mu_\tau) = A^T \frac{\partial}{\partial \nu} u(\nu,\mu_\tau)$$

Differenciating $\nabla_u \mathrm{L}\left(u(\nu,\mu_\tau),\nu,\mu_\tau\right)$ with respect to ν and using the equations above, we obtain that

$$\nabla^2_{\nu\nu}\mathrm{D}(\nu,\mu_\tau) = -A^T\left(\nabla^2_{uu}\mathrm{L}\left(u(\nu,\mu_\tau),\nu,\mu_\tau\right)\right)^{-1} A$$

Optimization Algorithm

Finally, the algorithm can be state as

$$u_{\tau+1} = \underset{u}{\operatorname{argmin}}\left\{ \mathrm{L}\left(u, v_{\tau}, \mu_{\tau}\right)\right\}$$

$$\mu_{\tau+1} = \max\left\{0, \mu_{\tau} + \rho\left(Cu_{\tau+1} - d\right)\right\}$$

$$v_{\tau+1} = v_{\tau} + \left[A^{T}\left(\nabla^{2}_{uu}\mathrm{L}\left(u_{\tau+1}, v_{\tau}, \mu_{\tau}\right)\right)^{-1} A\right]^{-1}\left(Au_{\tau+1} - b\right)$$

Optimization Algorithm

By considering the proposed optimization problem we obtain

ECPs in parallel

$$u_{i,\tau+1} = -\left(Q_i + \frac{I}{\rho} + C_i^T I^\rho C_i\right)^{-1}\left(q_i + \left[A^T \nu_\tau\right]_i + C_i^T \mu_{i,\tau} - \frac{u_{i,\tau}}{\rho} - C_i^T I^\rho d_i\right)$$

$$\mu_{i,\tau+1} = \max\left\{0, \mu_{i,\tau} + \rho\left(C_i u_{i,\tau+1} - d_i\right)\right\}$$

Communicate $u_{i,\tau+1}$ to the ECM

ECM

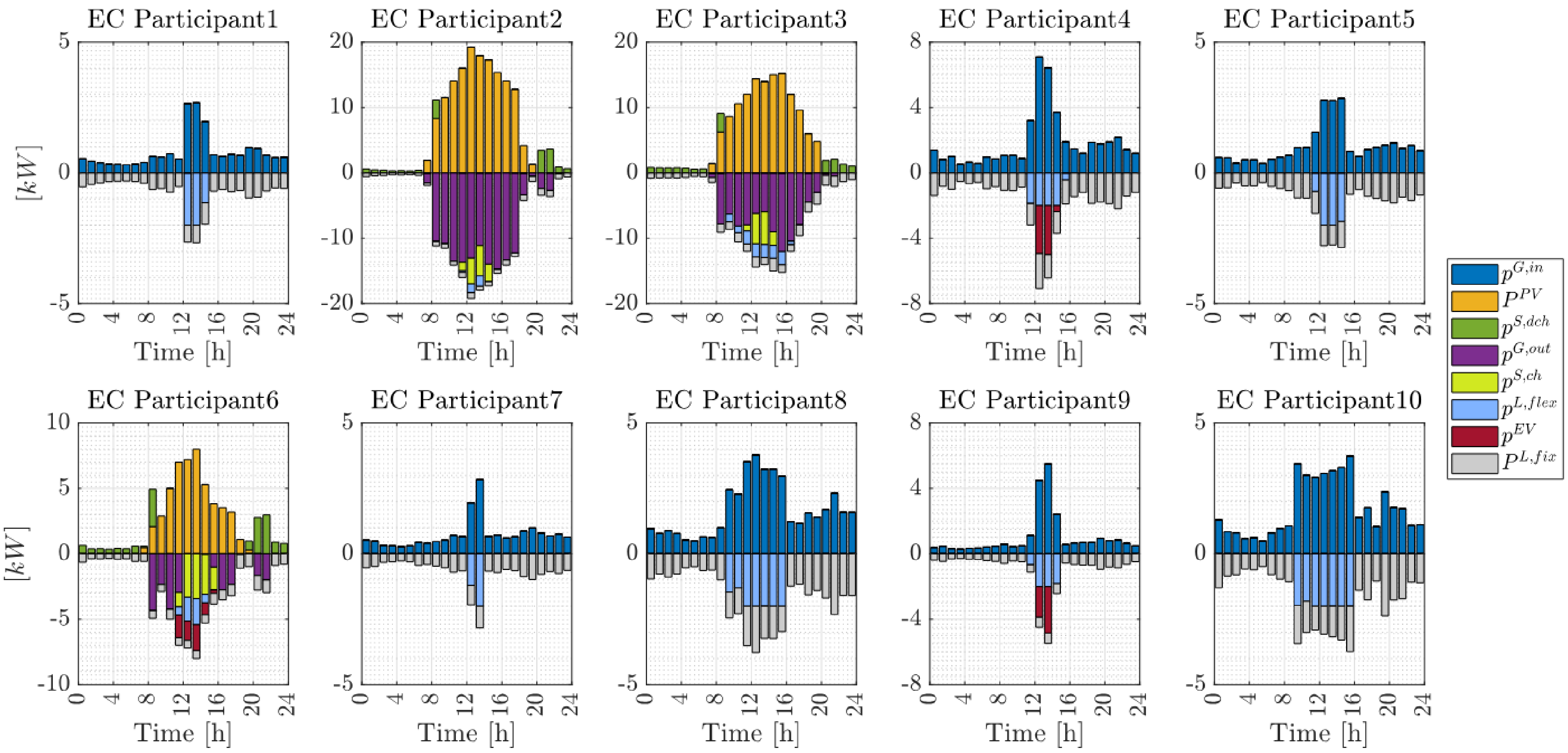
$$\nu_{\tau+1} = \nu_\tau + \left[A\left(\nabla_{uu}^2 \mathrm{L}\left(u_{\tau+1}, \nu_\tau, \mu_\tau\right)\right)^{-1} A^T\right]^{-1}\left(Au_{\tau+1} - b\right)$$

Communicate $\nu_{\tau+1}$ to the ECPs.

Case Study

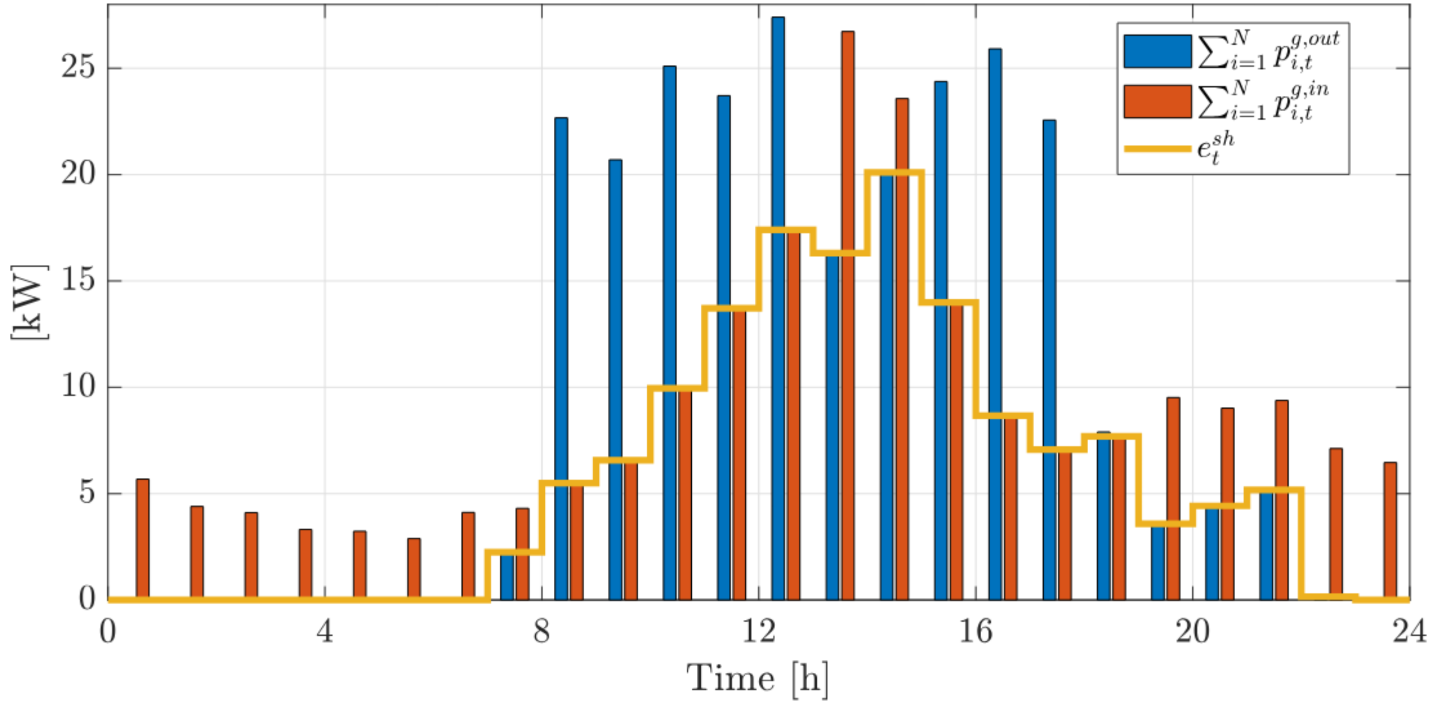
	CAP^S	X_0^S	$\overline{P}^S$	$E^{l,flex}$	CAP^{EV}	T^{EV}
ECP1	/	/	/	3.21	10	8
ECP2	30	0.2	10	7.01	10	11
ECP3	20	0.2	10	11.31	/	/
ECP4	/	/	/	8.31	10	16
ECP5	/	/	/	6.56	/	/
ECP6	15	0.2	10	5.14	10	21
ECP7	/	/	/	3.20	/	/
ECP8	/	/	/	12.76	/	/
ECP9	/	/	/	6.50	10	13
ECP10	/	/	/	13.78	/	/

Case Study



EC Participant1
EC Participant2
EC Participant3
EC Participant4
EC Participant5
EC Participant6
EC Participant7
EC Participant8
EC Participant9
EC Participant10
[kW]
Time [h]
$p^{G,in}$
P^{PV}
$p^{S,dch}$
$p^{G,out}$
$p^{S,ch}$
$p^{L,flex}$
p^{EV}
$P^{L,fix}$

Case Study



Total shared energy

140.41 kWh

Incentive

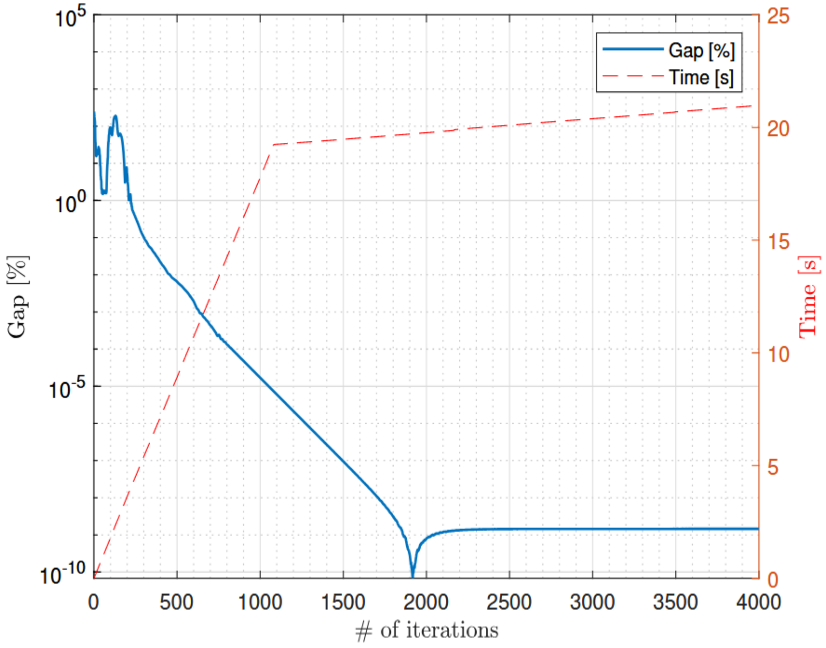
15.44€

Total cost of energy

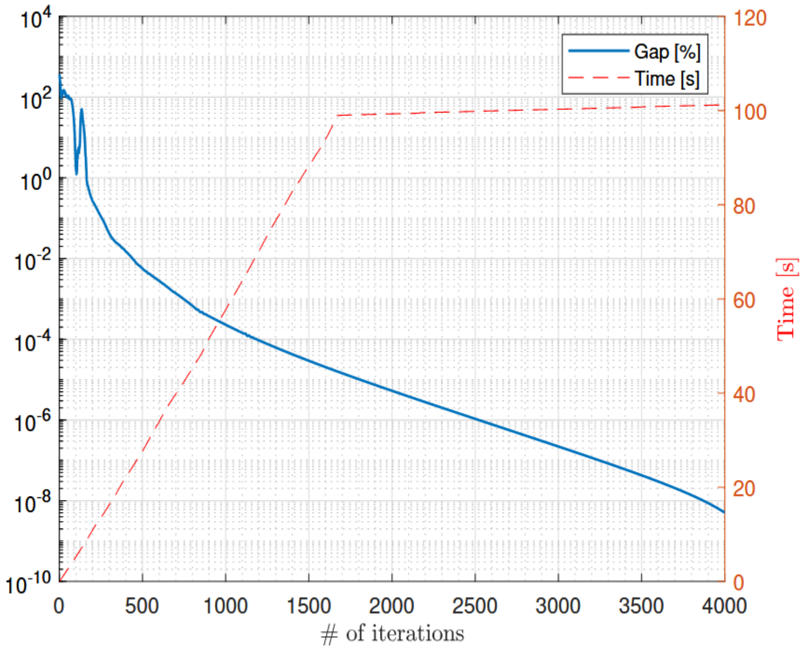
96.53€

Case Study

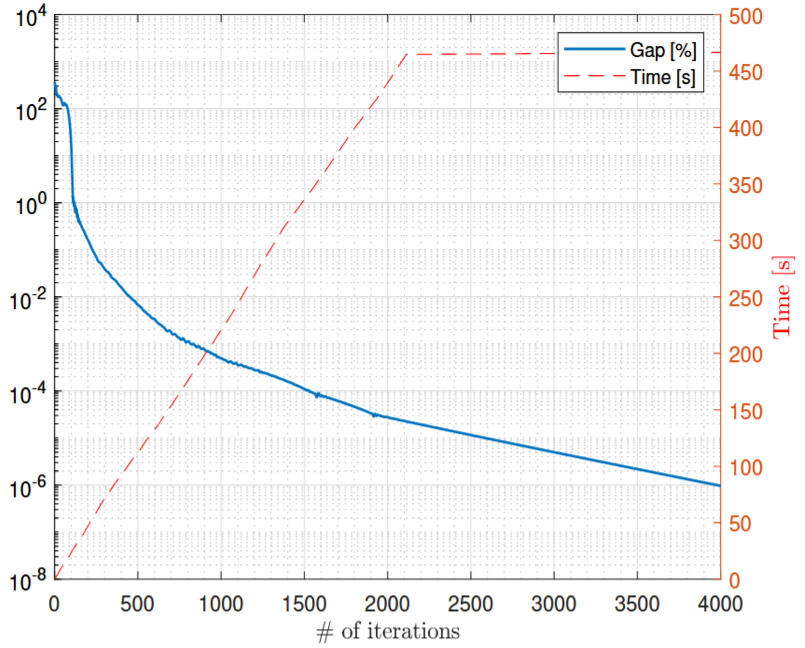
10 ECPs



20 ECPs



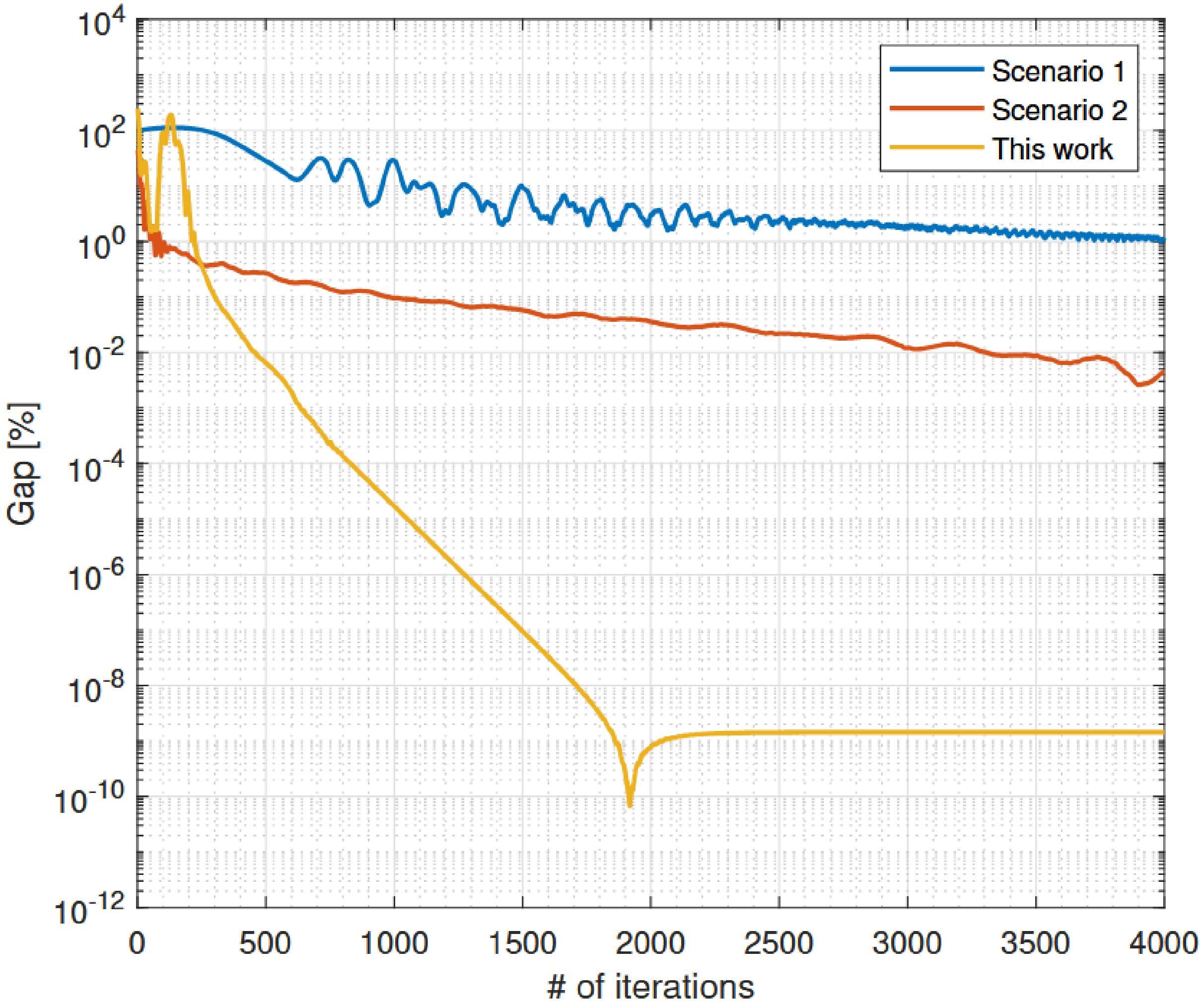
50 ECPs



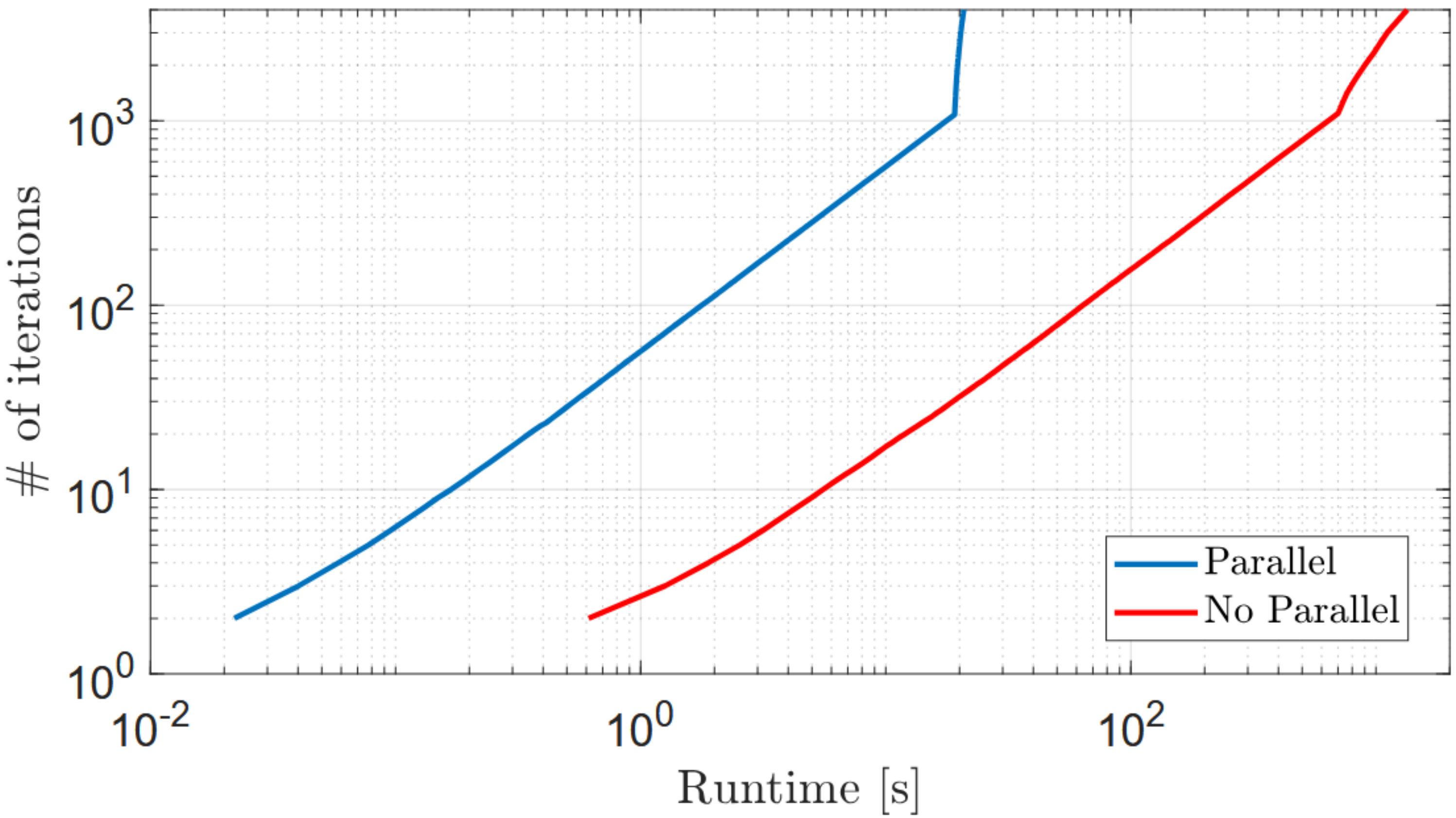
Case Study

Scenario 1:
Gradient-based dual update

Scenario 2:
Gradient-based dual update +
Augmented Lagrangian



Case Study



# of iterations
10^0
10^1
10^2
10^3
10^-2
10^0
10^2
Runtime [s]
Parallel
No Parallel

DISTRIBUTED OPTIMIZATION FOR ENERGY MANAGEMENT

FIRST ORDER APPROACHES

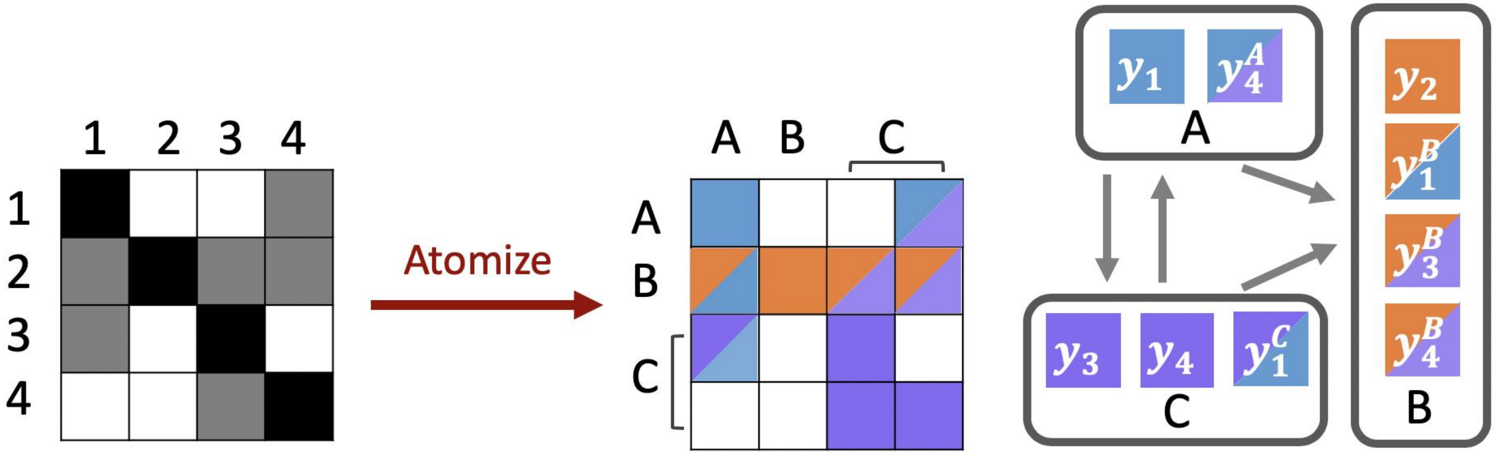
PAC algorithm decomposition

Global Network Problem → K Local Problems

$$\min_{y} \sum_{i=1}^{S} f_i(y) \quad \text{s. t.} \begin{cases} Gy = b \\ Hy \le d \end{cases}$$

$$\min_{a_j} \sum_{j \in K} f_j(a_j) \quad \text{s. t:} \begin{cases} G_j a_j = b_j, & j \in K \\ H_j a_j \le d_j, & j \in K \\ B_j a = 0, & j \in K \end{cases}$$

Each $[B]_j$ being adjacency matrix with edges $\left(y_k \to y_k^j\right)$



$$L_j\left(a_j, \mu_j, v, a_j \in Ha_j \le d_j\right) = f_j(a_j) + \mu_j^T\left(G_j a_j - b_j\right) + \sum_{i \in A(j)} v_i^T B_{ji} a_i$$

LOCAL PROBLEM LAGRANGE FUNCTION

*Romvary, J. J., **Ferro, G.**, Haider, R., & Annaswamy, A. M. (2021). A proximal atomic coordination algorithm for distributed optimization. IEEE Transactions on Automatic Control, 67(2), 646-661.*

PAC algorithm statement

$$a_j[\tau+1] = \underset{a_j \in \mathbb{R}^{|a_j|}}{\arg\min} \left\{ L_j\left(a_j, \bar{\mu}_j[\tau], \bar{\nu}[\tau]\right) + \frac{1}{2\rho}\left\| a_j - a_j[\tau] \right\|_2^2 \right\}$$

REGULARIZED PRIMAL UPDATE

$$\mu_j[\tau+1] = \mu_j[\tau] + \rho\gamma_j\left(\tilde{G}_j a_j[\tau+1] - b_j\right)$$

$$\bar{\mu}_j[\tau+1] = \mu_j[\tau+1] + \rho\hat{\gamma}_j[\tau+1]\left(\tilde{G}_j a_j[\tau+1] - b_j\right)$$

$$Comunicate\ a_j\left[\tau+1\right]\ \ to\ \forall i \in A(j)$$

FEASIBILITY DUAL UPDATE

$$\nu_j[\tau+1] = \nu_j[\tau] + \rho\gamma_j \sum_{i \in A(j)} B_{ji} a_i\left[\tau+1\right]$$

$$\bar{\nu}_j[\tau+1] = \nu_j[\tau+1] + \rho\hat{\gamma}_j[\tau+1] \sum_{i \in A(j)} B_{ji} a_i\left[\tau+1\right]$$

$$Comunicate\ \bar{\nu}_j[\tau+1]\ \ to\ \forall i \in A(j)$$

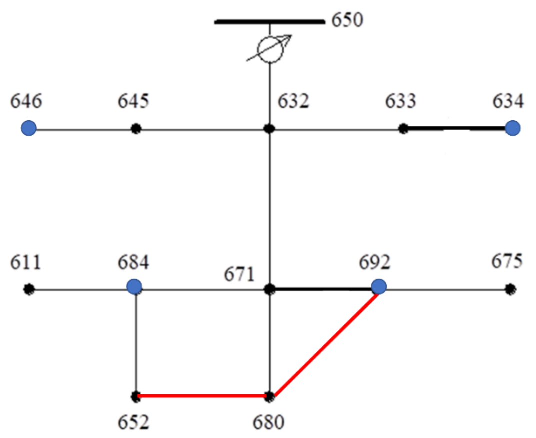
COORDINATION DUAL UPDATE

*Romvary, J. J., **Ferro, G.**, Haider, R., & Annaswamy, A. M. (2021). A proximal atomic coordination algorithm for distributed optimization. IEEE Transactions on Automatic Control, 67(2), 646-661.*

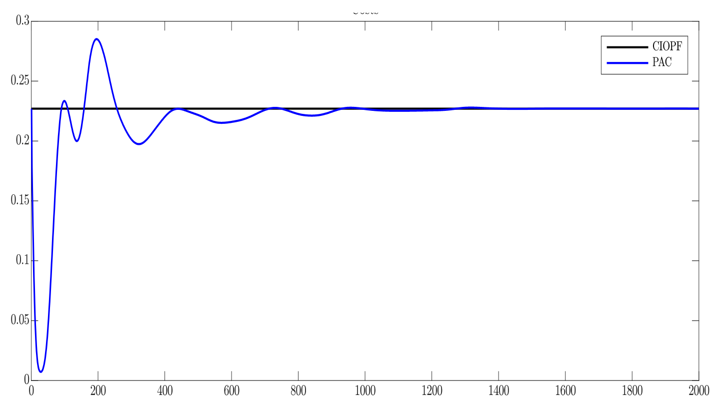
Applications: PAC to the OPF problem

POWER NETWORK (NONLINEAR/CONVEX)

POWER NETWORK (NONLINEAR/NONCONVEX)



PAC



Ferro G, Robba M, D'Achiardi D, Haider R, Annaswamy A. A distributed approach to the Optimal Power Flow problem for unbalanced and mesh networks. IFAC-PapersOnLine. 2020;53(2):13287-13292. doi:10.1016/j.ifacol.2020.12.159

Contributions

- A new convex relaxation of the OPF problem based on MCE, that can model distribution grids that may be unbalanced, or have a meshed topology.
- The development of a distributed algorithm, based on the PAC method, with clear articulation of conditions for proof of convergence

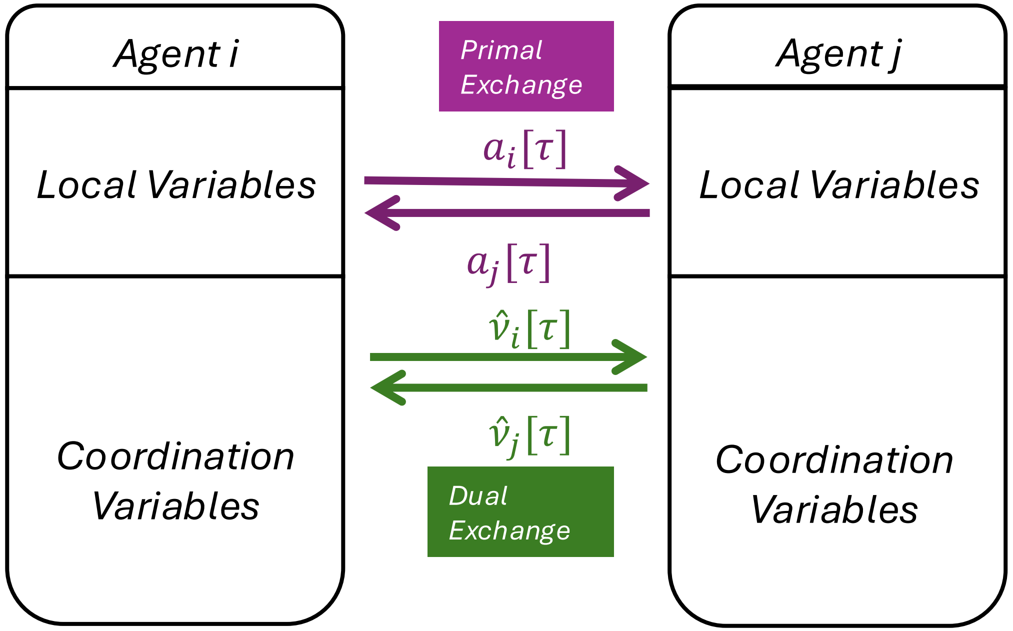
Data privacy in PAC

A rogue agent **j** may seek to utilize information about agent **k** to recover the information about the trajectory of $\boldsymbol{\nu_k}$

PROTECTION OF ANY SENSITIVE INFORMATION MEANS PRIVACY

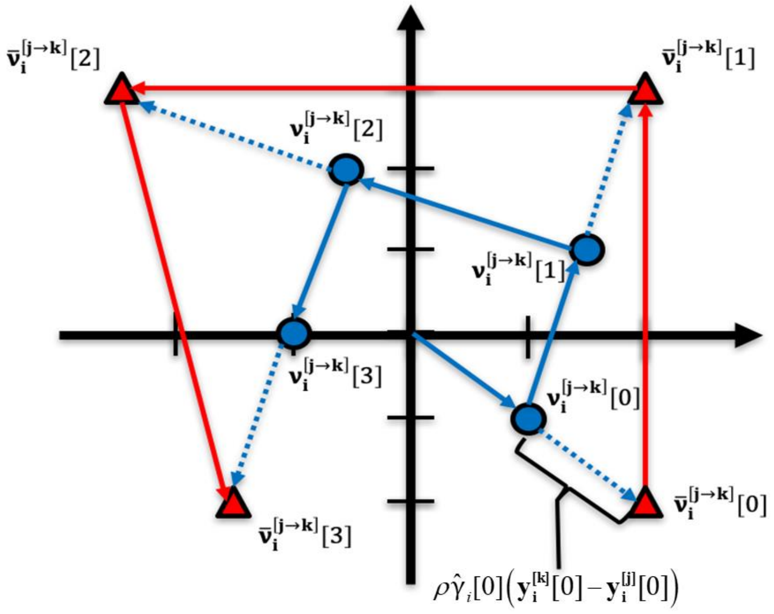
STATE OF THE ART ALGORITHMS **DO NOT** PRESERVE PROVACY SINCE COORDINATION DUAL VARIABLES ARE BROADCASTED

EXCHANGE OF VARIABLES AT ITERATION τ



PAC exchanges a "masked" dual variable $\hat{\nu}$, keeping real value ν private

Data privacy in PAC



The ***TRUE*** value of the coordination constraint is given by the BLUE circles and the $v_i^{[j\to k]}[\tau]$ points.
The ***PROTECTED*** value of the coordination constraint is given by the RED triangles and is communicated from *j* to *k*.
In this configuration, agent *k* only sees the RED trajectory of the ***PROTECTED*** value and not the trajectory BLUE of the ***TRUE*** value.

PAC LIMITS

- NO primal variables privacy
- Slow convergence due to iteration varying parameters

SOLUTION → **NESTEROV ACCELERATION**

First order acceleration methods

Polyak Heavy-ball method:

$$x_{k+1} = x_k - \alpha \nabla f(x_k) + \beta(x_k - x_{x-1})$$

which can be rewritten using PAC-flavour notation as:

$$x_{k+1} = \hat{x}_k - \alpha \nabla f(\hat{x}_k)$$
$$\hat{x}_{k+1} = x_{k+1} + \beta(\hat{x}_k - \hat{x}_{k-1})$$

where we can further consider the primal optimization of PAC:

$$x_{k+1} = argmin \left(\mathcal{L}(x, \mu_k, \nu_k) + \frac{1}{2\rho} \|x - \hat{x}_k\|_2^2 \right)$$
$$\hat{x}_{k+1} = x_{k+1} + \beta\left(\hat{x}_k - \hat{x}_{x-1}\right)$$

Nesterov acceleration:

$$y_{k+1} = x_k - \frac{1}{L} \nabla f(x_k)$$
$$x_{k+1} = y_{k+1} + \beta\left(y_{k+1} - y_k\right)$$

which can be rewritten using PAC-flavour notation as:

$$x_{k+1} = \hat{x}_k - \frac{1}{L} \nabla f(\hat{x}_k)$$
$$\hat{x}_{k+1} = x_{k+1} + \beta(x_{k+1} - x_k)$$

where we can further consider the primal optimization of PAC:

$$x_{k+1} = argmin \left(\mathcal{L}(x, \mu_k, \nu_k) + \frac{1}{2\rho} \|x - \hat{x}_k\|_2^2 \right)$$
$$\hat{x}_{k+1} = x_{k+1} + \beta\left(x_{k+1} - x_k\right)$$

NST PAC

$$a_j[\tau+1] = \underset{a_j \in \mathbb{R}^{|T_j|}}{\operatorname{argmin}} \Big\{ \mathcal{L}_j\left(a_j, \hat{\mu}_j[\tau], \hat{\nu}[\tau]\right) + \frac{\rho_j\gamma_j}{2}\|G_j a_j - b_j\|_2^2 + \frac{\rho_j\gamma_j}{2}\|B_j a_j\|_2^2 + \frac{1}{2\rho_j}\|a_j - a_j[\tau]\|_2^2 \Big\}$$

$$\hat{a}_j[\tau+1] = a_j[\tau+1] + \alpha_j[\tau+1](a_j[\tau+1] - a_j[\tau])$$

$$\mu_j[\tau+1] = \hat{\mu}_j[\tau] + \rho_j\gamma_j(G_j\hat{a}_j[\tau+1] - b_j)$$

$$\hat{\mu}_j[\tau+1] = \mu_j[\tau+1] + \phi_j[\tau+1](\mu_j[\tau+1] - \mu_j[\tau])$$

Communicate $\hat{a}_j$ for all $j \in [K]$ with neighbors

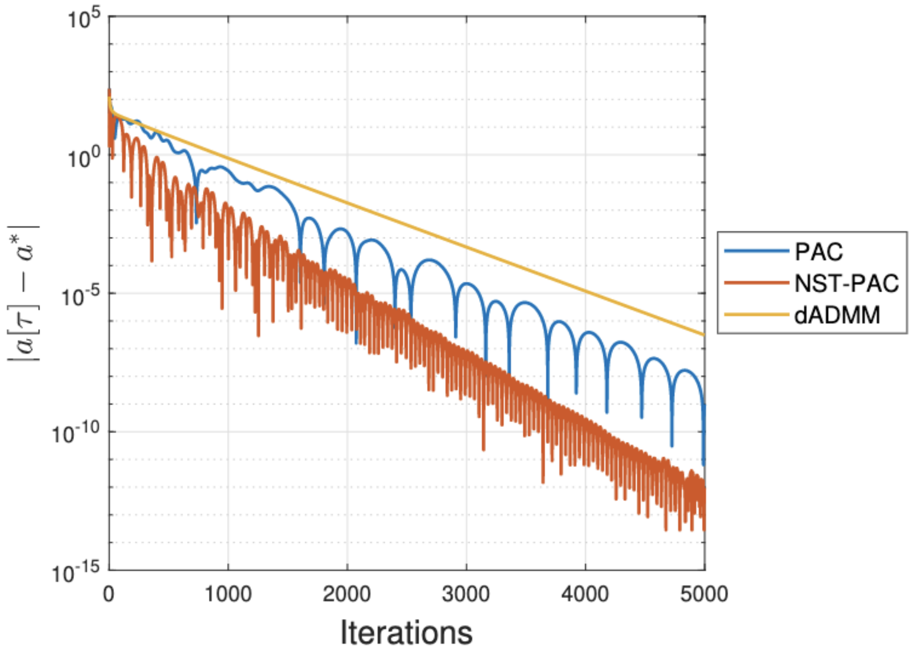
$$\nu_j[\tau+1] = \hat{\nu}_j[\tau] + \rho_j\gamma_j B_j\hat{a}_j[\tau+1]$$

$$\hat{\nu}_j[\tau+1] = \nu_j[\tau+1] + \theta_j[\tau+1](\nu_j[\tau+1] - \nu_j[\tau])$$

Communicate $\hat{\nu}_j$ for all $j \in [K]$ with neighbors

ACCELERATED AND PRIVATE PRIMAL/DUAL EXCHANGE

COMPARISON WITH STATE OF THE ART APPROACHES



***Ferro, G.**, Robba, M., Haider, R., & Annaswamy, A. M. (2022). A distributed optimization based architecture for management of interconnected energy hubs. IEEE Transactions on Control of Network Systems.*

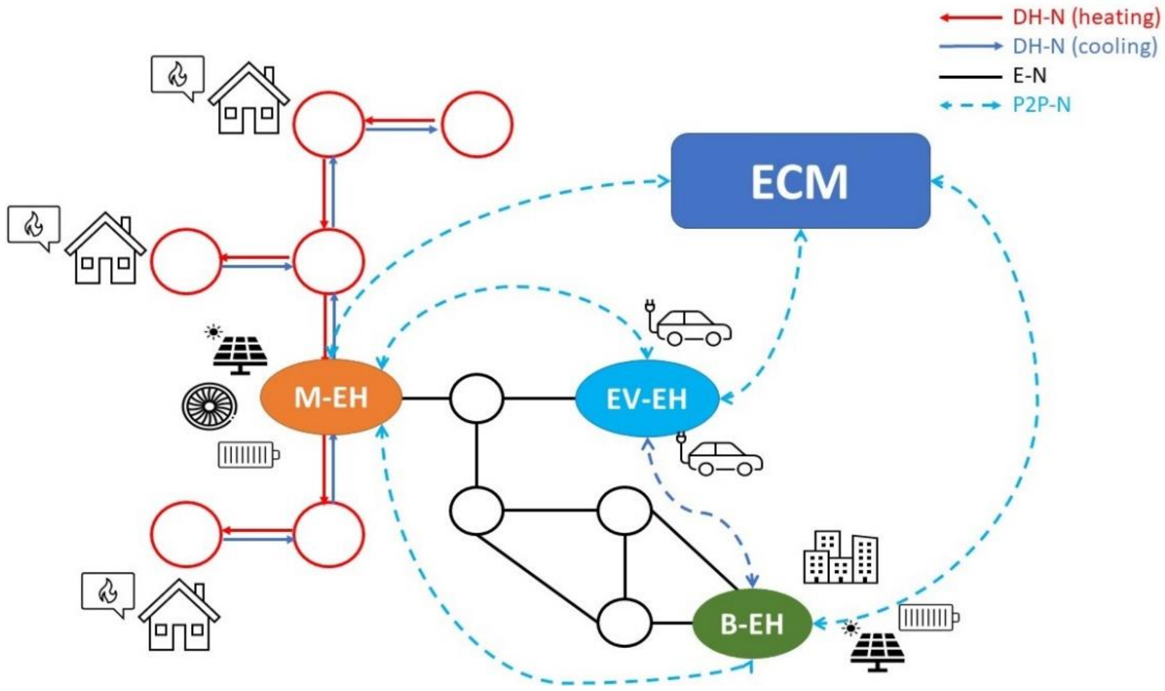
Application: NST-PAC to interconnected networks

ENERGY HUBS (EH)

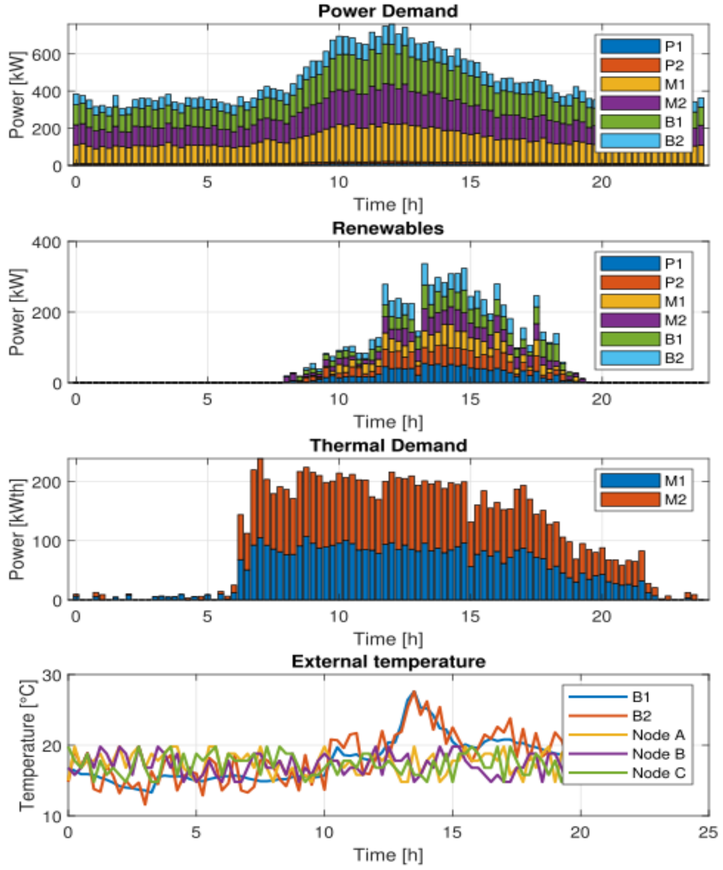
- Microgrid based Energy Hubs (M-EH)
- Building based Energy Hubs (B-EH)
- Electric Vehicles based Energy Hubs (EV-EH)

COUPLED NETWOKRS

- Distribution Electric Network (E-N)
- District Heating Network (DH-N)
- Peer to Peer Network (P2P-N)



Application: NST-PAC to interconnected networks



***Ferro, G.**, Robba, M., Haider, R., & Annaswamy, A. M. (2022). A distributed optimization based architecture for management of interconnected energy hubs. IEEE Transactions on Control of Network Systems.*

Contributions

- Definition of a systems architecture that manages interconnected energy hubs in a smart city using an ECM
- Development of new PAC based distributed optimization algorithm with Nesterov acceletration and full privacy properties.
- Coupled management of power (E-N) and district heating (DH-N) networks through a peer to peer energy exchange

IEEE 13 BUS

Cost	Baseline	Our Approach	Cost reduction %
E-N	28896.3	27257.8	-5.67
DH-N	4743.7	2228.65	-53.03

IEEE 123 BUS

Cost	Baseline	Our Approach	Cost reduction %
E-N	2219.37	2007.88	-9.7
DH-N	1100.81	346.75	-68.49

SECOND ORDER APPROACHES

Algorithms' acceleration: second order methods for ALM (AL-SODU)

ONE OF THE MAIN CAUSES OF SLOW CONVERGENCE IN DISTRIBUTED OTIMIZATION IS DUAL UPDATE

$$\min_{x \in \mathbb{R}^n} \quad f(x)$$
$$s.t. \, Ax = b$$

$$\mathcal{L}(x, \mu) = f(x) + \mu^T (Ax - b)$$

Gradient descent for dual update

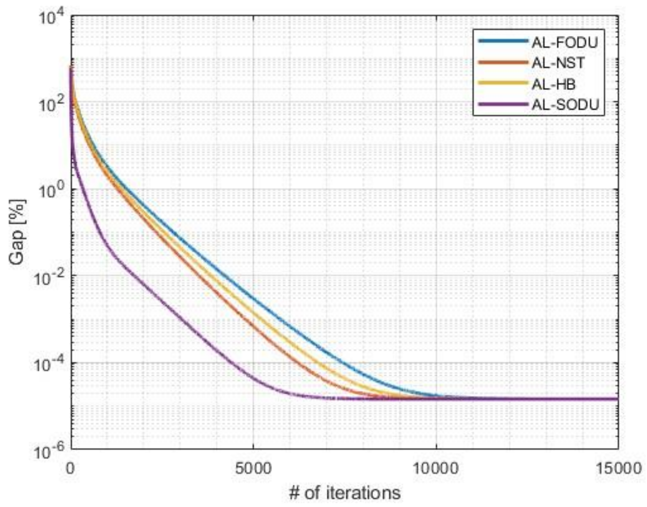
Algorithm:

$$x_{k+1} = argmin \;\; \mathcal{L}(x, \mu_k)$$
$$\mu_{k+1} = \mu_k + \rho(Ax - b)$$

Second-order Dual Update

Algorithm:

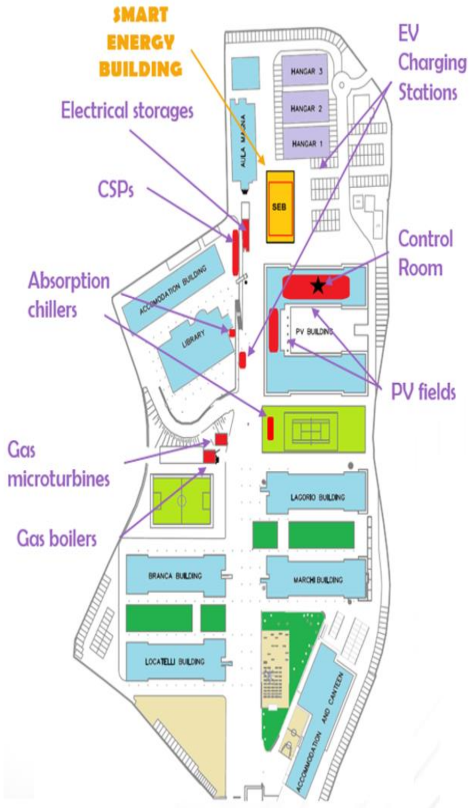
$$x_{k+1} = argmin \;\; \mathcal{L}(x, \mu_k)$$
$$\mu_{k+1} = \mu_k + [A^T(\nabla^2_{xx}\mathcal{L}(x_{k+1}, \mu_k))\mathrm{A}]^{-1}$$
$$[Ax - b]$$

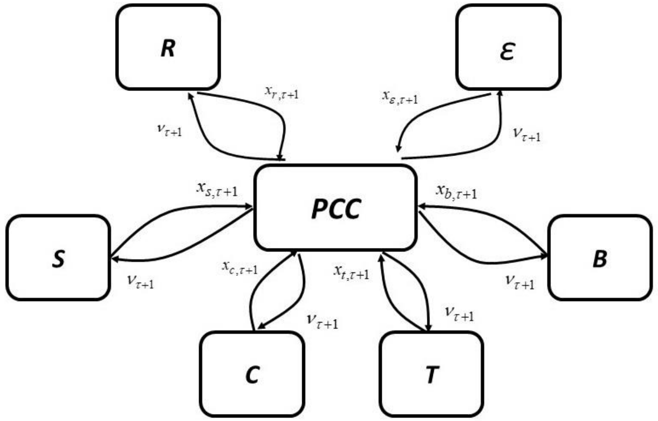


G. Ferro, M. Robba, F. Delfino, R. Haider, A. M. Annaswamy, "Distributed operational management of microgrids: a second order dual update approach," IFAC (2023)

Application: AL-SODU to microgrid management

The EMS optimization problem can be **solved in parallel amongst each component**.





- Renewable units *R*;
- Storage system (e.g. batteries) *S*;
- Cogenerative plants *C*;
- Thermal plants (e.g. boilers) *T*;
- Smart building *B*;
- Electric vehicles charging park *E;*
- Point of common coupling (PCC)

$$x_{\tau+1} = -\left(Q_i + \frac{1}{\rho}\right)^{-1} \left(q_i^{\mathrm{T}} + [A^{\mathrm{T}}\nu_\tau]_i + C_i^{\mathrm{T}}\mu_{i,\tau} - \frac{x_{i,\tau}}{\rho}\right), \; i \in \mathcal{A}$$

PRIMAL UPDATE

$$\mu_{i,\tau+1} = \max\left\{0, \mu_{i,\tau} + \rho\left(C_i x_{i,\tau+1} - d_i\right)\right\}, \; i \in \mathcal{A}$$

Communicate $x_{i,\tau+1}$ $\forall i \in \mathcal{A}$ to the $\mathcal{P}$ node

PROJECTION ON FEASIBILITY SPACE

$$\nu_{\tau+1} = \nu_\tau + \left[\left(A^{-1}\right)^{\mathrm{T}}\left(Q + \frac{1}{\rho}\right)A^{-1}\right]^{-1}\left(Ax_{\tau+1} - b\right)$$

Communicate $\nu_{\tau+1}$ to $i \in \mathcal{A}$.

SODU

Ferro, G., Robba, M., Haider, R., & Annaswamy, A. M. (2025). A Second-Order Dual Update Approach for the Decentralized Optimal Scheduling of Polygenerative Microgrids. *IEEE Transactions on Control Systems Technology*.

Algorithms' acceleration: second order methods for ADMM (Nwt-ADMM)

THE IDEA IS TO APPLY THE **SECOND ORDER UPDATE** TO THE POPULAR ALTERNATING-DIRECTION METHOD OF MULTIPLIERS (**ADMM**) ALGORITHM

$$\min_{x,z} f(x) + g(z) \quad s.t. \quad Ax + Bz = c$$

AUGMENTED LAGRANGIAN

$$L_\rho(x, z, \lambda) = f(x) + g(z) + \lambda^T (Ax + Bz - c) + \frac{\rho}{2} \|Ax + Bz - c\|_2^2$$

ADMM

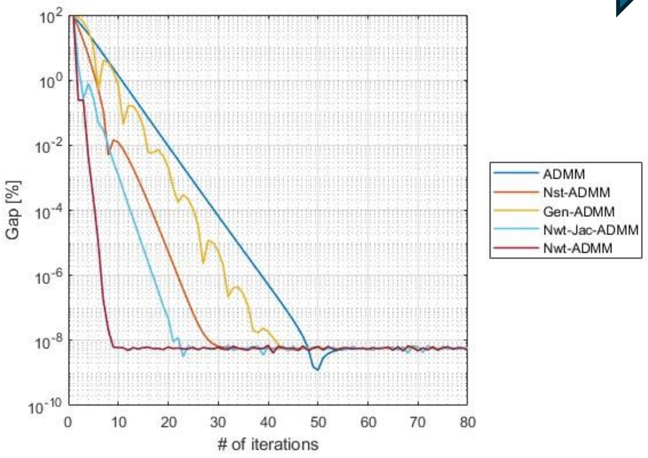
$$x_{k+1} = \underset{x}{\text{argmin}}\, L_\rho(x, z_k, \lambda_k)$$
$$z_{k+1} = \underset{z}{\text{argmin}}\, L_\rho(x_{k+1}, z, \lambda_k)$$
$$\lambda_{k+1} = \lambda_k + \rho\,(Ax_{k+1} + Bz_{k+1} - c)$$

Second-order Dual Update

$$\lambda_{k+1} = \lambda_k + M\left(x_{k+1}, z_{k+1}, \lambda_k\right)^{-1} (Ax_{k+1} + Bz_{k+1} - c)$$

$$M\left(x_{k+1}, z_{k+1}, \lambda_k\right) = \begin{bmatrix} A & B \end{bmatrix} \mathbf{J}_{x,z}\left(F\left(x_{k+1}, z_{k+1}, \lambda_k\right)\right)^{-1} \begin{bmatrix} A^T \\ B^T \end{bmatrix}$$

$$\mathbf{J}_{x,z}\left(F\left(x, z\right)\right) = \begin{bmatrix} \nabla^2_{xx} f\left(x\right) + \rho A^T A & \rho A^T B \\ \rho B^T A & \nabla^2_{zz} g\left(z\right) + \rho B^T B \end{bmatrix}$$



Aicardi, M., & **Ferro, G**. (2024). A Newton-Based Dual Update Framework for ADMM in Convex Optimization Problems. IEEE Control Systems Letters.

THANK YOU FOR YOUR ATTENTION

Virginia Casella

virginia.casella@edu.unige.it

Luca Parodi

luca.parodi@edu.unige.it

Giulio Ferro

giulio.ferro@edu.unige.it